

Math in Focus™: Theory and Practice

Volume 2

In the last edition of *Math in Focus: Theory and Practice*, we discussed transition and offered suggestions to help with transitioning into a new curriculum, scaffolding students into the more rigorous tasks and helping parents understand your curriculum's new pedagogy. One of the transitions that has made the biggest difference for Singapore and its students' understanding is instructing content with a concrete-pictorial-abstract approach. Read on to learn more about the importance of this instructional model.

Concrete-Pictorial-Abstract: In Theory

Using concrete materials to introduce mathematical concepts is not new. The idea of creating understanding by beginning with a concrete object and moving to a drawing of the concept before addressing the abstract representation of the idea makes perfect sense. Let's take a look at what it means.

Concrete

In the concrete stage, the **teacher** begins instruction by modeling each mathematical concept with concrete materials (e.g., two sided counters, cubes, base-ten blocks, pattern blocks, fraction bars, or geometric solids) and **students represent problems with concrete materials**. The importance of students taking over the responsibility of creating the mathematical situations cannot be emphasized enough.

Pictorial

In this stage, the **teacher and student** transform the concrete model into a **pictorial** (semi-concrete) level, which may involve drawing pictures using circles, dots, rectangles, tallies, etc. Initially, the picture looks exactly like the concrete material then can become more abstract. The students will recognize a picture before being able to create the picture. But, until the student can create the mathematical situation pictorially, true understanding will not happen.



Abstract

At this stage, teachers and students record the **mathematics concept at a symbolic level**, using only numbers, notation, and mathematical symbols to represent the concept. Operation symbols (+, -, \times , \div) to indicate addition, subtraction, multiplication, or division and symbolic representations of amounts are used. At this level, true understanding of concepts can be seen.

Concrete-Pictorial-Abstract: In Practice

As mentioned above, instruction with a concrete-pictorial-abstract progression is not a new idea. However, implementing such a structure into a classroom can, at times, not run as smoothly as intended nor yield the results one wants. Unless the student has moved through each phase, true understanding is not developed. Even a student who demonstrates math content at the abstract level may not actually have understanding. Examples of this lack of understanding include (but are not limited to) a student who can count on to add, but would count on 9 more in the problem $3 + 9$; a student who can use the algorithm to solve a 3-digit subtraction problem, but does not understand why using it to solve $300 - 299$ is not efficient; a student who can solve long division problems, but could not show/explain it with concrete materials.

In theory, we can understand the importance of moving through each phase and would take the time to give students these experiences, but in practice, there can be roadblocks. Here are a few questions teachers have raised regarding the CPA Pedagogy and suggestions for moving forward:

1. My students have always used manipulatives, but I've always kept them in a big tub so that students feel like they can get them when they need them:

Sometimes manipulatives can be seen as “toys” instead of mathematical representations of relationships or situations. If manipulatives are dumped into a tub and not given a purpose, they become useless as mathematical tools. Children should be allowed to explore, but this time should be limited and quickly replaced with purposeful mathematics.

After a few minutes to explore, have students help you bag manipulatives into useful configurations: Individual Bags with a set number of cubes, Base 10 Blocks, etc., Table Bags for sharing with 2 or more students, etc. Make a poster with the number of manipulatives that should be in each bag and set a time limit for completing this task. Be prepared for human error and have a few extras on hand.





2. My students are too disruptive to use manipulatives. They don't treat them with respect and it can interfere with the lesson:

We have to think carefully about how we will approach classroom management when using manipulatives. Here are some suggestions:


- Choose only the manipulative students need for the lesson and have them packaged and ready for the lesson. No exploratory time needed in this case!
- Use the first 10-15 minutes of class time to show how to take them out of the bag, put them back into the bag, pass them to a neighbor, etc.
- Send students back to their seats for 3-4 minutes (or away from the group) during the lesson if the manipulative isn't being used properly.
- As a reward, allow students to play with them at the end if they are used correctly.
- Also, use a bag for every two students so that it's more manageable.


3. I taught with the manipulatives, but my students still don't show understanding OR how long do I allow them to use the manipulatives?

Developing conceptual understanding is an individual process. Some students might be able to represent situations easily with concrete materials and then move on to drawing those same relationships, some may be able to represent the situations with concrete materials, but can only recognize similar drawings instead of creating them him/herself, and others may study and use the manipulatives until you give them a reason to move forward. The important thing to keep in mind is that each stage is a necessary part of building understanding. If one skips the pictorial and tries to tie concrete materials to the abstract, students may initially show understanding, but will not have the foundation for applying the understanding later. Equally, only using teacher drawings or pictorial representations from the book, will not prove that the students have created their own visual and thus leave the abstract representation weak. Allow students to use concrete materials until they can create the pictorial representation successfully. And remember, the journey to understanding is not linear so students may waver between stages.

4. What about my students who already know the abstract rule? How will I know whether they really understand?

One time, when showing a long division lesson from grade 4, a teacher said, "Oh, my students already know how to divide, if I show them this, it will totally mess them up!" It was only a split second before she realized what that meant and she said, "Wow! I need to back up!" Teachers of intermediate grades, when adopting *Math in Focus* and its pedagogy, struggle (as do teachers of primary grades) with the difficulty and rigor of the new standards. However, they have the added difficulty of asking students to prove their answers concretely or pictorial when all the students know is the procedure. The students can usually get to the answer, but have no understanding of how or why it is the answer and that doesn't seem to bother them.





This procedural knowledge typically got them through elementary school and didn't cause a problem until they were presented with non-routine problem solving situations and algebraic reasoning. This will no longer be the case with the new standards.

It is imperative that you ask students to build, create, draw and/or explain for all situations even if the book doesn't ask them to (remember, it is written on the assumption that the understanding has already been built). It is not necessary to have them do it for every problem perhaps, but a few on each assignment so that you have a record of their understanding. Use the Transition Guide to find the grade level that the understandings are developed concretely and pictorially, and introduce your students to some of those lessons/games/hands-on activities to begin the understanding.

In closing, we discussed that the concrete-pictorial-abstract approach impacted Singapore instruction and student understanding. The same can be true for our schools and students! As you are implementing *Math in Focus*, we understand that using the concrete-pictorial-abstract progression of teaching might seem challenging to manage at times, but the impact on student achievement and teacher success will be well worth your efforts.

Hope your implementation is going well-

The *Math in Focus* Specialist Team

