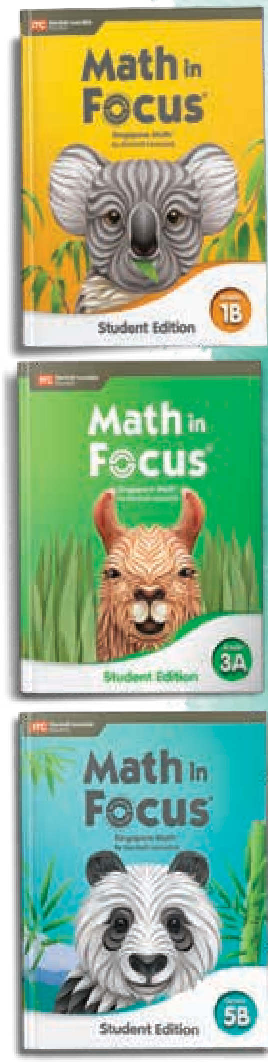
HMH

Math in Focus®
Singapore Math® by Marshall Cavendish®

Algebra in the Elementary Grades: How *Math In Focus* Addresses Algebraic Thinking

Why Algebra?



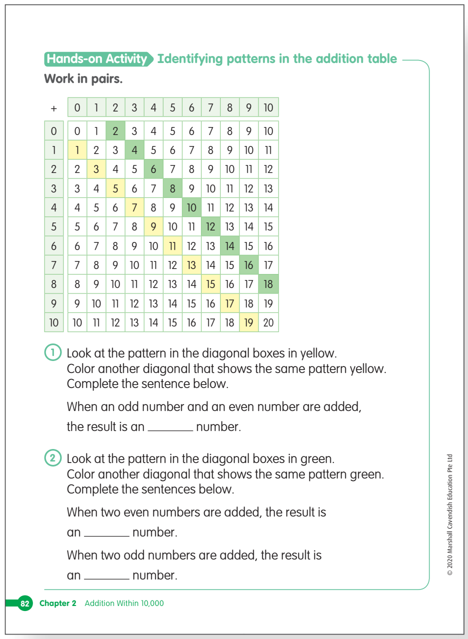
Educators have been increasingly interested in the transition from arithmetic to algebra, accompanied by an interest in algebraic thinking in the elementary grades. This interest has been amplified because so many students seem to have trouble learning algebra. It is not uncommon to hear from adults that they did fine in mathematics until it become about the last three letters of the alphabet. While some people define algebra as "generalized arithmetic," it in fact is a very different way of thinking than merely numerical or computational arithmetic. It is a system of logical reasoning. It is a representational system, involving manipulation of symbols, not numbers, and a subject of study in mathematics. It is about structures and relationships.

For example, if a student is given the equation $2x + 5 = 15$, a good arithmetic student can solve this mentally, not by finding an equivalent equation that isolates the unknown. Perhaps a better strategy would be to ask the student to write the equation that represents the question, "5 more than double a number is 15, what is the number?" This question focuses on the representation of a relationship using a variable, and not on the particular numbers.

To address the lack of success in algebra, math educators and policy makers recommend that the transition from arithmetic to algebra be more explicit and introduced in the earliest grades. ***Math In Focus*®*: Singapore Math*® *by Marshall Cavendish*® © *2020*** addresses this need in two ways—first by making sure students have sufficient understanding and fluency with critical topics such as whole number computation, fractions, ratios, and proportionality, and second by explicitly teaching critical algebraic ideas and reasoning. Some of the topics explicitly taught include generalizations, recognizing structures, properties, equivalence, use of variables, and creating visual models to solve algebraic problems.

Generalizations

Mathematics is the study of generalizations—of finding a relationship that applies across a set of objects. It requires looking across objects and cases for similarities, differences, and definable commonalities. It is leading to abstraction, to a stripping of context.


Hands-on Activity Identifying patterns in the addition table

Work in pairs.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

1. Look at the pattern in the diagonal boxes in yellow.
Color another diagonal that shows the same pattern yellow.
Complete the sentence below.

   When an odd number and an even number are added, the result is an ________ number.

2. Look at the pattern in the diagonal boxes in green.
Color another diagonal that shows the same pattern green.
Complete the sentences below.

   When two even numbers are added, the result is an ________ number.

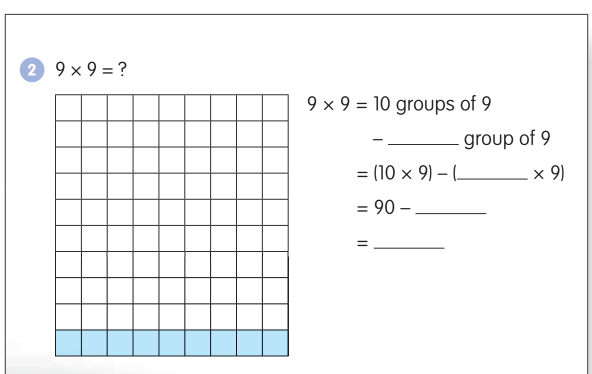
   When two odd numbers are added, the result is an ________ number.

82 Chapter 2 Addition Within 10,000

For example, students in the beginning of third grade are asked to consider the effects of adding an even number to an even number, an odd number to an odd number, and an even number to an odd number. After trying several versions of each of these, students are asked if that relationship is always true. Students often will provide additional examples as proof that an even sum results when the two addends are even or when both are odd. The teacher is then suggested to ask *why* this relationship holds, which forces students to move away from additional examples and instead focus on the meaning of even and odd numbers. What do all even numbers have in common and what do all odd numbers have in common? The use of manipulatives encourages students to see even numbers as pairs and odd numbers as pairs plus 1. In later grades this will become $2n$ and $2n+1$. Students will discuss if 0 is an even number, then later in the year, students will look at the multiplication table in the same way and make generalizations about the products of even and odd numbers. They will answer the questions of whether there are more even or odd products on the table and why.

Structures

When the multiplication facts are taught as related facts and students learn facts by using these relationships, they are introduced to structures and properties.


2 $9 \times 9 = ?$

9×9 = 10 groups of 9

– ________ group of 9

= (10×9) – (________ $\times$ 9)

= 90 – ________

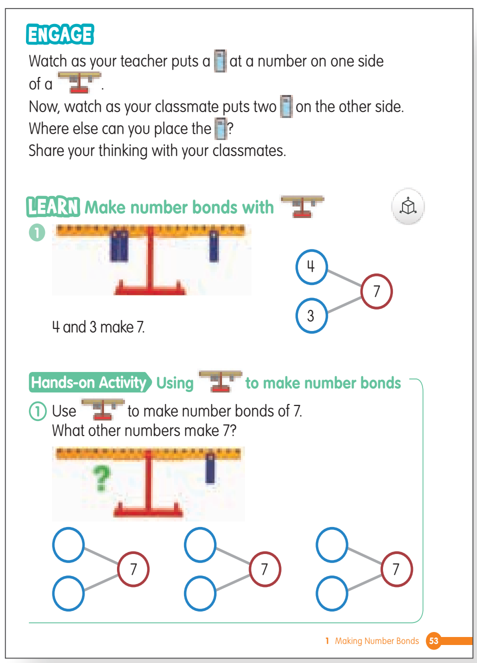
= ________

For instance, students are encouraged to use an area model to learn the facts for 9 using the related facts for 10. To learn 9 x 9, students already know 10 groups of 9 equals 90, so just one less group of 9 would equal 81. Similarly, students learn that 9 times any number is one less group than 10 times a number. This will be used later for problems like 9 x 15 is (10 x 15) – 15. Eventually this will become $9n = 10n - n$. Some students may prefer to break down 9 x 9 to be (5 x 9) + (4 x 9), which introduces the distributive property. So much mental arithmetic depends on this distributive property and is needed in the expansion and factoring of variable expressions. The focus is not on the technical names of properties but on an understanding that structures like the associative, commutative, and distributive property enable both number and symbol manipulation.

Equivalence

Equivalence is a key algebraic concept. Equivalence is the concept that any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value. From the beginning of kindergarten and first grade, students are taught in *Math in Focus* that the equal sign means that the quantities on both sides of the equal sign are equal or the same amount.

This example is from Chapter 2 of first grade. To make clear that numbers can be composed and decomposed to make number bonds and equations, student use an actual balance beam to decompose the number 7 into two parts. Students experiment to see the difference of quantities that balance or are level and those that are not.

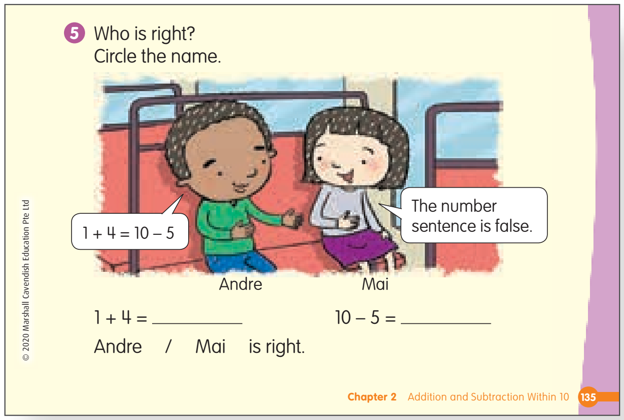


As students experiment, the teacher asks how many ways students can make the balance level with 7 on one side. The teacher might put the 7 on either the left or the right side. When students learn to write equations using number bonds, they may record their findings as 7 = 3 + 4 or 5 + 2 = 7.

The teacher may ask questions such as:

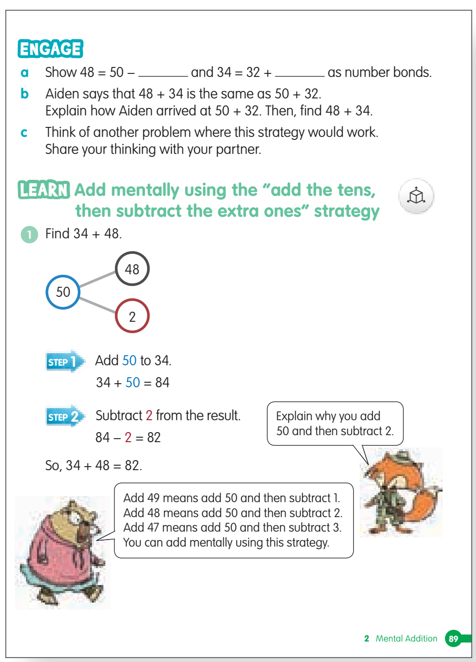
- What are some other names for 7?
- How many ways can you make 7?
- Why does 4 + 3 = 3 +4?
- Why does 4 + 3 = 1 + 6?
- 5 is how many less than 7?

The balance beam reinforces the concept that the amount on both sides of the equal sign is the same. Notice also that we are beginning to introduce the idea of a variable—what are two numbers that equal 7? To make the concept of equivalence even stronger, students are presented with problems like 5 + 4 =__+ 6 and 2 + 4 =__+ 1. The equal sign shifts from meaning "the answer" to finding an equivalent or equal amount on both sides.



As students discuss which person is correct in the example above, they use the concept of equivalence, or "same amount," to defend their reasoning. While the focus in Chapter 2 is on addition and subtraction facts to 10, the Essential Question of the Chapter is "How can you tell if a number sentence is true or false?"

Equivalence plays a role in every grade level. In second grade, students learn regrouping so they can represent 156 as 1 hundred, 5 tens, and 6 ones; as 1 hundred, 4 tens, and 16 ones; or as 0 hundreds, 15 tens and 6 ones, and so on. In third grade, students use equivalence to mentally add and subtract two- and three-digit numbers.



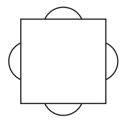
Students use number bonds to explicitly represent equivalence. In fourth grade, students find equivalent expressions for mixed numbers, and in fifth grade, students find equivalent expressions for fractions so they have common denominators. In sixth grade, students expand and factor expressions to see if they are equivalent, and find equivalent ratios. In seventh grade, students solve equations with variables on both sides by finding equivalent expressions, and in eighth grade, students solve systems of equations by finding equivalent expressions to solve by elimination.

Functions

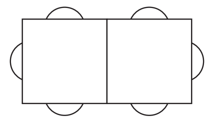
Students are introduced to functional thinking from the beginning of kindergarten, where they learn to count, to the end of eighth grade, where they learn what a function is and multiple ways of representing a function. Even by fourth grade, students use functional thinking to solve problems by considering linear growing patterns, as in this example:

3. A square table can seat 4 people.
How many square tables are needed to seat 26 people if the tables are put together?

Hint:



1 table can seat 4 people.



2 tables can seat 6 people.

Students use many strategies to solve this problem. They look for a pattern 4, 6, 8, 10, and so on. They recognize that each table adds two people. They count the people on each side of the table. They subtract 2 from the 26 for the end people and then divide 24 into 2. They use a variety of strategies that all lead to generalizations for any number of people.

Variables

The concept of a variable is also challenging but introduced early in first grade. Part of the reason the concept of a variable is challenging is that it is used in a variety of related but different ways. In some cases, a variable represents a set of numbers, as in the expression $2n$. In other cases, the variable represents a single unknown, as in $2x = 10$. In another case, variables are used to describe a relationship or function, as in $y = 2x + 5$. Finally, while we tell students that a variable may be any symbol, when it is used in a formula, such as $A = ½\,bh$, the specific letters do matter and stand for parts of the triangle.

Throughout *Math In Focus*, students are provided opportunities to use variables in all these contexts. In first grade students are presented with this Think problem:

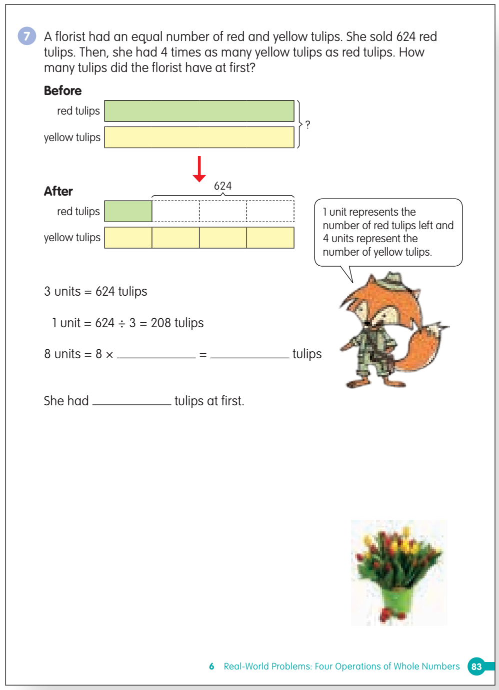
THINK

Karina sees some monkeys in a zoo.
The zookeeper tells her there are fewer than 15 monkeys.
Karina writes the addition sentence to show the number of monkeys.

10 + ________ = ________

Talk about some of the likely answers with your partner.

Students are asked to consider all the numbers that would satisfy the condition and the blank lines are used to represent all those numbers.



7 A florist had an equal number of red and yellow tulips. She sold 624 red tulips. Then, she had 4 times as many yellow tulips as red tulips. How many tulips did the florist have at first?

Before

red tulips

yellow tulips

?

After

624

red tulips

yellow tulips

1 unit represents the number of red tulips left and 4 units represent the number of yellow tulips.

3 units = 624 tulips

1 unit = 624 ÷ 3 = 208 tulips

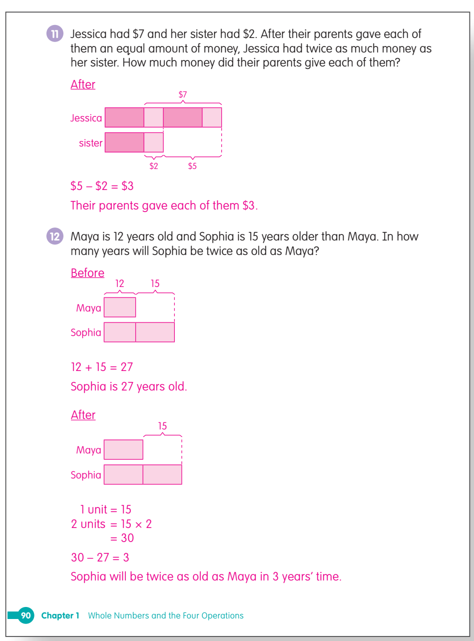
8 units = 8 × ____________ = ____________ tulips

She had ____________ tulips at first.

6 Real-World Problems: Four Operations of Whole Numbers 83

By fifth grade, after the introduction of bar models to represent variables in second and third grade, students are asked to solve more complex algebraic problems using bar models. Such a problem is shown above. While the algebra of this problem is ¾ *x* = 624, *x* = 832 for red tulips and 2 x 832 represents the original number of tulips, this is beyond fifth grade. Instead students use visual models, called bar models, to represent the situation. Notice the students use the bars to represent two situations—before and after. Initially there are an equal number of red and yellow tulips, but we don't know that quanitity. Students draw two bars of equal length to represent the quantity. We do know that the yellow ones remaining after 624 red ones are sold are four times as much as the remaining red tulips. So there are 5 units left, and 3 of those units equal 624. One unit equals 208, and 8 units equal 1,664.

In another example, students are given the information that Maya is 12 and her sister Sophia is 15 years older. They are asked when Sophia will be twice as old as Maya.



11 Jessica had $7 and her sister had $2. After their parents gave each of them an equal amount of money, Jessica had twice as much money as her sister. How much money did their parents give each of them?

After

$7

Jessica

sister

$2 $5

$5 − $2 = $3

Their parents gave each of them $3.

12 Maya is 12 years old and Sophia is 15 years older than Maya. In how many years will Sophia be twice as old as Maya?

Before

12 15

Maya

Sophia

12 + 15 = 27

Sophia is 27 years old.

After

15

Maya

Sophia

1 unit = 15

2 units = 15 × 2

= 30

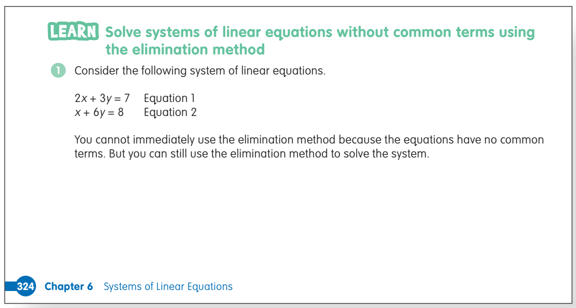
30 − 27 = 3

Sophia will be twice as old as Maya in 3 years' time.

90 Chapter 1 Whole Numbers and the Four Operations

Students can solve this problem numerically by guessing and checking, but the focus in this lesson is on representing the relationships with models or visual variables. Students again draw one model for the before, revealing that Sophia is 15 years older and therefore is 27 years old. In the after representation, Sophia is twice as old as Maya, but the difference in their ages must still be 15 and 30 is twice 15. Students see that they can subtract 27 from 30 to see in how many years she will be double Maya's age. Taken further, students try additional differences in ages to see that the number of years will always be the difference of the number of years older one person is minus the age of the younger person.

In these examples, the concept of a variable and relationships between quantities are represented with bar models that can easily become symbolic variables. In sixth grade, students solve positive variable equations. In seventh grade, students solve problems with rational coefficients and variables on both sides. In eighth grade, students solve systems of linear equations. Even when solving sophisticated problems, reference is made to the bar models, which now include the use of *x* and *y* to represent quantities.



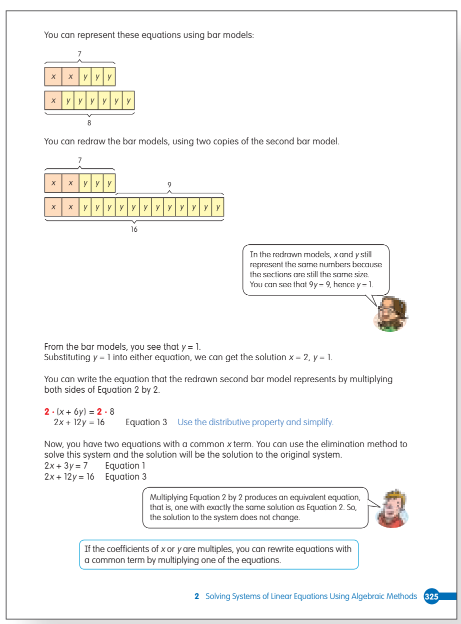
LEARN Solve systems of linear equations without common terms using the elimination method

1 Consider the following system of linear equations.

$2x + 3y = 7$ Equation 1
$x + 6y = 8$ Equation 2

You cannot immediately use the elimination method because the equations have no common terms. But you can still use the elimination method to solve the system.

324 **Chapter 6** Systems of Linear Equations



You can represent these equations using bar models:

You can redraw the bar models, using two copies of the second bar model.

From the bar models, you see that $y = 1$.
Substituting $y = 1$ into either equation, we can get the solution $x = 2$, $y = 1$.

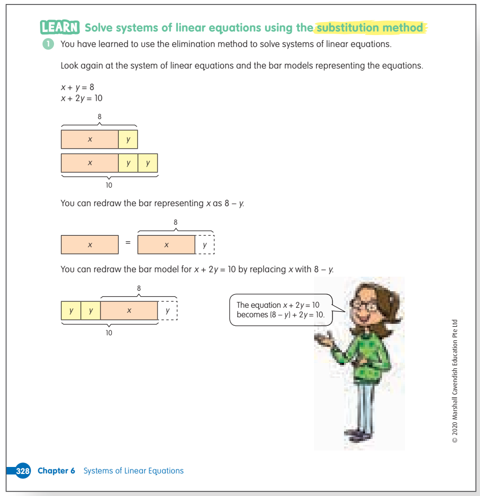
You can write the equation that the redrawn second bar model represents by multiplying both sides of Equation 2 by 2.

$\mathbf{2} \cdot (x + 6y) = \mathbf{2} \cdot 8$
$2x + 12y = 16$ Equation 3 Use the distributive property and simplify.

Now, you have two equations with a common *x* term. You can use the elimination method to solve this system and the solution will be the solution to the original system.
$2x + 3y = 7$ Equation 1
$2x + 12y = 16$ Equation 3

If the coefficients of *x* or *y* are multiples, you can rewrite equations with a common term by multiplying one of the equations.

2 Solving Systems of Linear Equations Using Algebraic Methods 325



LEARN Solve systems of linear equations using the substitution method

1 You have learned to use the elimination method to solve systems of linear equations.

Look again at the system of linear equations and the bar models representing the equations.

$x + y = 8$
$x + 2y = 10$

You can redraw the bar representing x as $8 - y$.

You can redraw the bar model for $x + 2y = 10$ by replacing x with $8 - y$.

Students learn to use both substitution and elimination to solve systems of equations using the combination of bar models and variable expressions.

The transition from arithmetic to algebra is challenging, but if students are introduced to algebra concepts throughout the elementary grades, that transition is made easier. *Math In Focus* addresses key concepts such as making generalizations, looking for consistent structures such as properties, recognizing equivalence, using variables and variable expressions, and representing algebraic problems with visual models. Teachers are encouraged to ask questions such as: "Is this always true? How do you know? How can you represent this relationship?" Questions like this help students move away from specific arithmetic examples and into generalization and common structures.

If math is taught with understanding and not as a set of procedures, students become used to looking for generalizations and structures. In the words of Jerome Bruner, "Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully. To learn structure in short, is to learn how things are related." Students who think of mathematics this way are more able to understand the structures that are introduced in algebra.

About the Author

Andy Clark

Math in Focus Grades 1–Course 3 U.S. Consultant
and Course 2–Course 3 Writer

Andy Clark is an advisor on *Math In Focus: Singapore Math®* by Marshall Cavendish® programs, which introduced the renowned Singapore curriculum to the United States. He is the former K–12 Math Director for Portland Public Schools, which outperformed the state and closed the achievement gap in mathematics. As director, he was principal investigator of the last Urban Systemic Grant authorized by the National Science Foundation. He has taught all levels of the K–12 system and has conducted professional development in more than 40 states. He is the author of *Algebra Readiness; Summer Success: Math; Partner Games 6; and Practice Counts Grades 1–6*.

Houghton Mifflin Harcourt.
The Learning Company™

hmhco.com