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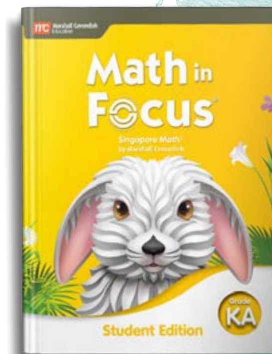
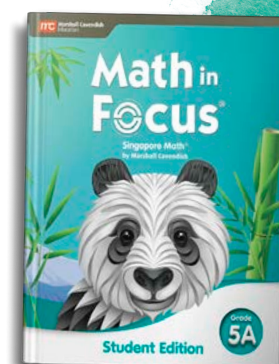
Math in Focus®
Singapore Math® by Marshall Cavendish®

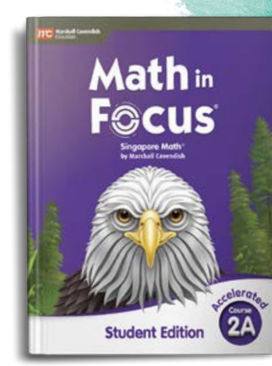
Math in Focus® supports teachers in developing students' relational understanding of mathematics.

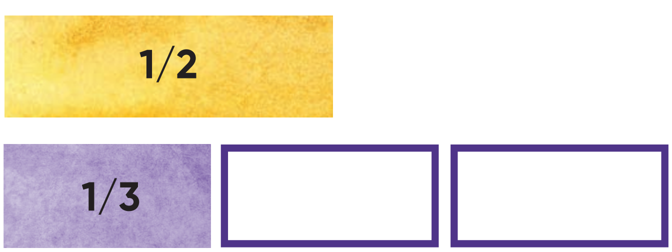
Richard Skemp, one of the math educators whose theory influenced the design of *Singapore Math*®, wrote about the difference between instrumental understanding and relational understanding. In the former, you are taught how to do something. In the latter, you are presented with tasks and explorations that build rich conceptual schemas that enable new ideas and connections to develop. For instance, telling students that the way to divide 1/3 by 1/2 is to invert 1/2 and multiply to get 2/3 creates instrumental understanding. Students get a correct answer but have no idea why and what they are doing. For relational understanding, students are presented with a series of problems such as 1 ÷ 1/2, 2 ÷ 1/2, 10 ÷ 1/2, 1/2 ÷ 1/3 and finally 1/3 ÷ 1/2. Students use paper circles to model the problems, and then draw pictures to find a pattern.

Students begin to understand that if they know how many pieces there are in 1 whole, they can figure out how many are in any part. If they know how many halves there are in 1 whole, they can figure out how many halves there are in 5 or 10 or any number of wholes. If they know how many 1/3 there are in 1, then they can multiply that to know how many 1/3 are in 2, in 1/2, or in 1/4. If they know how many 1/2 are in 1, they can multiply that by 1/3 to find how many 1/2 are in 1/3. Finally, they realize that in a fraction *a*/*b*, the number of *a*/*b* in 1 whole is always *b*/*a*, which we call the reciprocal. It is the definition of a reciprocal: what number times *a*/*b* equals 1? It is *b*/*a*.









How many 1/2 in 1/3?

There are two 1/2 in 1 whole so 1/3 x 2 in 1/3

1/3 ÷ 1/2 = 1/3 x 2 = 2/3 or 2/3 of a half in 1/3

What are the implications for developing relational understanding?

For students it means that the process is as important as the product. How we solve a problem is as important as the answer. Students must generalize, look for patterns, make connections, and develop fluency. They must learn to think mathematically. For teachers, it means structuring learning so that students have the time to build a schema that connects what they already know to the new concept. It means doing fewer problems but doing them thoroughly, including multiple strategies and connections. It means providing the concrete and visual models that enable students the access to even the most complex concepts. Finally, it means teachers must have a deep understanding of the mathematics underlying the math concepts. There is nothing simple about simple arithmetic, much less mathematics.

To give just one example, kindergarten teachers understand that basic counting of objects requires complex understanding that each object gets one label, and only one label. The final label tells you the total number of objects. Students also need to know there is no designated label one, or two, but once given a label, an object can't have another. The labels must be used in a specific order: one, two, three etc. Counting objects is much more complicated than merely reciting the counting sequence.

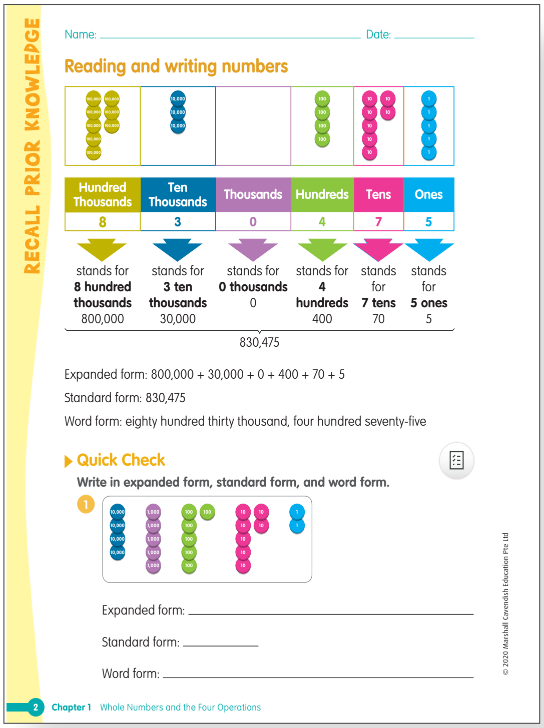
Sometimes teachers need help identifying the mathematics underlying the concept they are teaching.

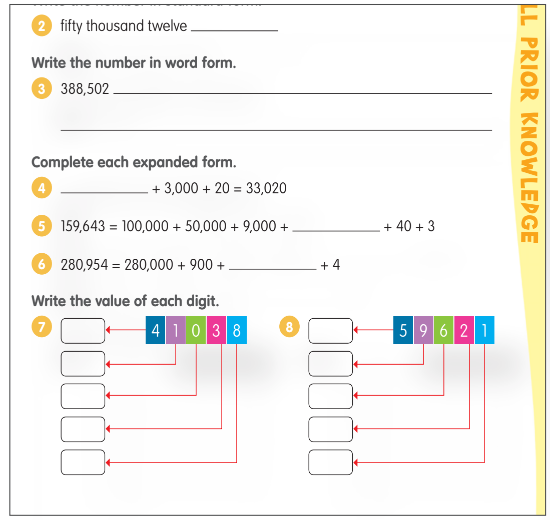
Moreover, they often need support to identify the prior concepts taught in previous years and the mathematics students will see in future grades. For instance second grade teachers must emphasize the concept of multiplication as equal groups and repeated addition. Students are introduced to the concept of arrays. Students are encouraged to use known array facts to determine unknown ones. For example if they know the 5 by 2 array is 10 then the 6 by 2 array must be 2 more or 12. But teachers must also be aware that in third grade students will be introduced to the area model of multiplication and that in fourth grade, multiplication will shift from repeated addition to comparison in preparation for fifth grade multiplication of fractions. The equation 30 = 6 x 5 means 30 is 5 times greater than 6, and 6 times greater than 5. Students will solve problems like, "Ann has $15 and Maria has 3 times as many dollars". Students begin to see that while they can add 15 three times, they can also think what number is 3 times greater than 15. This helps prepare students so that in fifth grade when students see 1/2 x 1/3, they can think of it as what number is 1/2 of 1/3? In sixth grade, students begin to deal with ratio and rate. Ratios can be part to part comparisons, while fractions are part to whole comparisons. Jerome Bruner, another major contributor to the Singapore Math® curriculum, described this as building new concepts on previously learned concepts, but always at a higher level.

Teachers can benefit from the strength of *Math In Focus* in that it provides the background, the progression, and most importantly, the access to these complex mathematical ideas.

Let's look at another example. In fifth grade students learn to multiply and divide by a two digit number fluently. In the past this was often taught mechanically and developed instrumental understanding. Sometimes the algorithm was not even taught with place value. Let's see how the concept is taught relationally in *Math In Focus*. The first question that teachers must ask is, "What is the math? What concepts are at the heart of the algorithm?". The algorithm is based on place value and the distributive property. That is, to multiply by 24 means to multiply by 4 and 20. Also critical is to recognize that to multiply 25 by 24 is the same as (4 x 5) + (4 x 20) + (20 x 5) + (20 x 20). Decomposing numbers is an essential concept. If these understandings are not firm, the algorithm may work but it won't be understood.

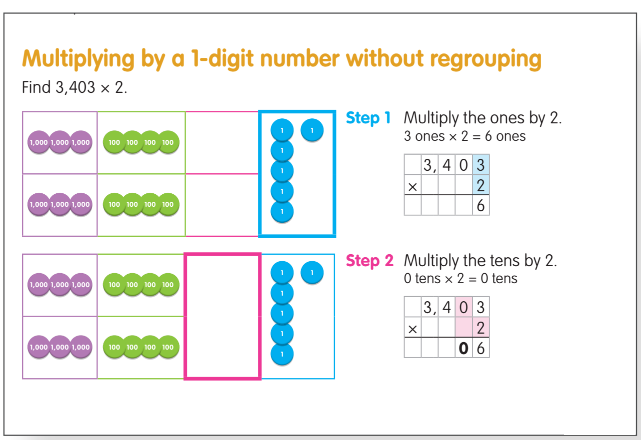
The second thing the teacher must consider is. What concepts will this be built on? This is sometimes called prior knowledge, but it is more than that. It really is, "What schema do students already have that can be built on to accommodate the new information?". These are laid out very carefully and intentionally in *Math In Focus*.

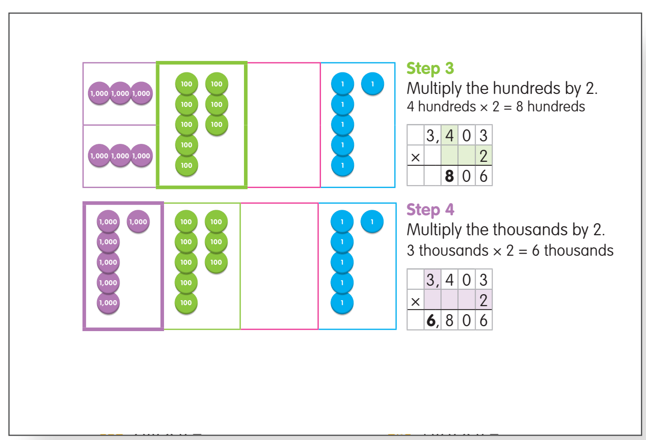




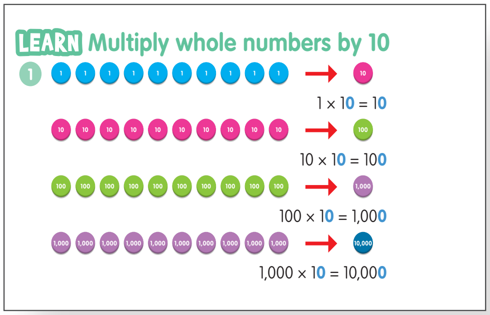
First, teachers are reminded to review the place value models presented in fourth grade. Students must understand that each place has a value; the value of a digit is determined by its relationship to the other digits. Each place to the left is ten times greater. These concepts are reinforced with place value chips and charts as well as pictorial models. Students are reminded that expanded form helps reinforce the place value concept and the ability to decompose numbers.

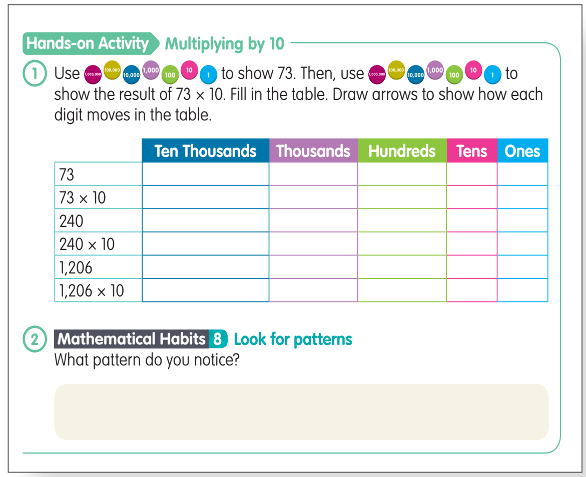
Next teachers are given an example from fourth grade of multiplying by a single digit. The concept is developed using place value chips and boards. Students physically show how to multiply the ones, the tens, the hundreds, and so on. Students compare an example that requires regrouping and one that doesn't.





Finally the teaching progression for learning multiplication by two digits multipliers is slowly and carefully sequenced. First, students use chips and diagrams to multiply whole numbers by ten and then by multiples of ten.





Similarly students multiply whole numbers by 100 and 1,000 and look for patterns based on place value: every place to the left is ten times greater. Students are given the following problem:

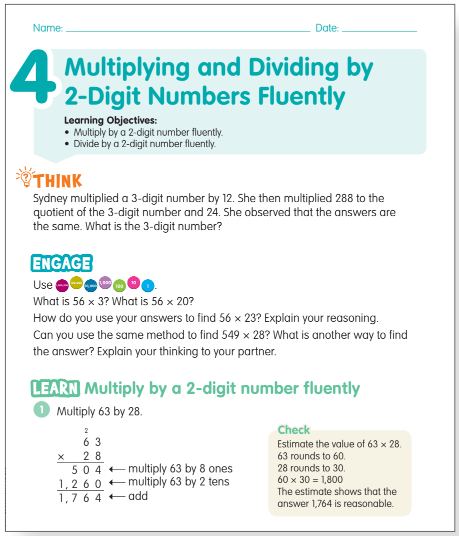
Using place value chips, answer: What is 12 x 100?; What is 12 x 300?; What is 12 x 1000?; What is 12 x 3000?; and finally, What does 12 x 50,000 equal?

Students are asked to explain their thinking and model the products using the place value chips. 12 x 100 is 12 hundreds or 1,200. 12 x 300 is 12 x 3 x 100. Similarly, since 12 x 1,000 is 12,000, then 12 x 3,000 is 12 x 3 x 1,000. Finally, 12 x 50,000 is 12 x 5 x 10,000. Imagine how different student understanding is from these examples rather than from simply counting zeros.

A distinct feature of *Math In Focus* is what Zoltan Dienes called "variation" as in the above problems. Practice items include this careful variation.

Students practice multiplying by multiples of 100 and 1,000 in the following sequence: 42 x 600, 42 x 6,000, and then 6 x 400 and 81 x 600. Next is 7,510 x 200 and 20 x 900. The next two examples are somewhat more complicated: 12 x 7,000 and 549 x 3,000.

By the time students are learning to use the paper and pencil model for multiplying by two digits, they are fluent in decomposing numbers and using place value to find products. Students work with partners to solve 56 x 3, and then 56 x 20. Students are asked how they can use this to solve 56 x 23, and that leads to an understanding that 56 x 23 requires adding the products of 3 x 56 and 20 x 56.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4 Multiplying and Dividing by 2-Digit Numbers Fluently

Learning Objectives:
- Multiply by a 2-digit number fluently.
- Divide by a 2-digit number fluently.

THINK

Sydney multiplied a 3-digit number by 12. She then multiplied 288 to the quotient of the 3-digit number and 24. She observed that the answers are the same. What is the 3-digit number?

ENGAGE

Use .
What is 56 × 3? What is 56 × 20?
How do you use your answers to find 56 × 23? Explain your reasoning.
Can you use the same method to find 549 × 28? What is another way to find the answer? Explain your thinking to your partner.

LEARN Multiply by a 2-digit number fluently

1 Multiply 63 by 28.

$$\begin{array}{rl} \overset{2}{6}\,3 & \\ \times\ \ 2\,8 & \\ \hline 5\,0\,4 & \leftarrow \text{multiply 63 by 8 ones} \\ 1,2\,6\,0 & \leftarrow \text{multiply 63 by 2 tens} \\ \hline 1,7\,6\,4 & \leftarrow \text{add} \end{array}$$

Check
Estimate the value of 63 × 28.
63 rounds to 60.
28 rounds to 30.
60 × 30 = 1,800
The estimate shows that the answer 1,764 is reasonable.

From kindergarten to eighth grade, *Math In Focus* develops generalizations and abstract concepts using concrete to pictorial to abstract models and variations in practice. Problems are sequenced so students are active learners, building new concepts on those learned prior. The process of solving problems is as important as the answers to the problems.

The last two decades of math research have concluded that we are still not teaching for understanding. Decades of cognitive research and educational experience have shown that when specific responses to specific tasks or questions are learned by rote, that knowledge does not generalize. According to the CRESST study done at UCLA, "Despite the robustness of this finding, however, mathematics instruction in the U. S. has historically focused on the memorization of specific responses to specific questions, and as a result, the knowledge most students have is extremely context bound and not generalizable."

Richard Skemp described the difference between instrumental understanding and relational understanding with a metaphor.

A person moves into a new town and must learn the route from home to work. Explicit instructions are given. The person will know the specific route. But if the person gets lost, it might be difficult to find his or her way back. That is instrumental understanding. If the person wants to know about the town, the person must roam the town, finding different routes, making missteps, even getting lost before finding a familiar street. The person now has learned the town and many routes to interesting places. If the person is in a hurry, then it is probably best to be told the route from here to there. But if the goal is to learn about the town, to get the big picture, then relational understanding is more useful. Our goal is to teach students the whole city that is mathematics. *Math In Focus* provides the support that teachers need to help students develop relational understanding.

About the Author

Andy Clark

Math in Focus Grades 1–Course 3 U.S. Consultant
and Course 2–Course 3 Writer

Andy Clark is an advisor on *Math In Focus®: Singapore Math® by Marshall Cavendish®* programs, which introduced the renowned Singapore curriculum to the United States. He is the former K–12 Math Director for Portland Public Schools, which outperformed the state and closed the achievement gap in mathematics. As director, he was principal investigator of the last Urban Systemic Grant authorized by the National Science Foundation. He has taught all levels of the K–12 system and has conducted professional development in more than 40 states. He is the author of *Algebra Readiness; Summer Success: Math; Partner Games 6;* and *Practice Counts Grades 1–6*.

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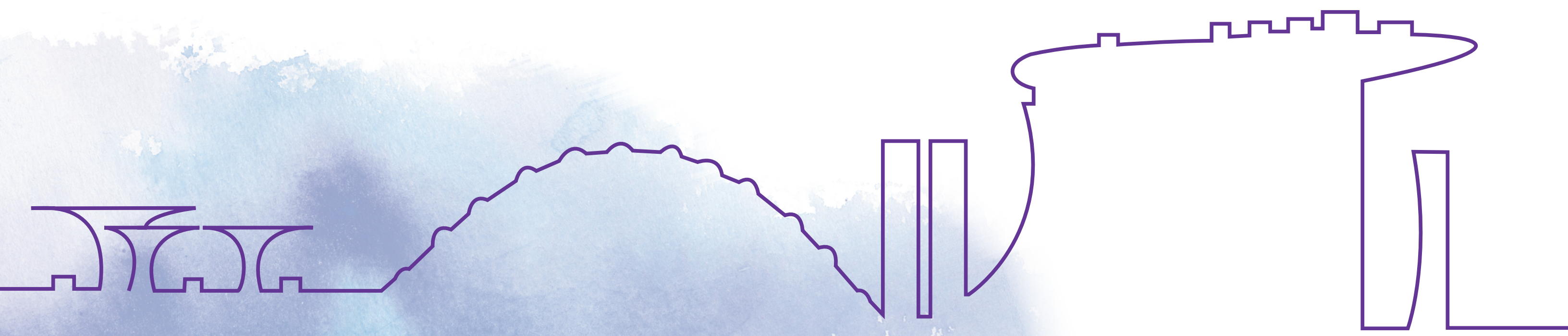

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Elevated Learning.
Proven Achievement.

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