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math expressions

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Pre-K Research Background and Design

RESEARCH BACKGROUND AND DESIGN OF *MATH EXPRESSIONS* EARLY LEARNING RESOURCES FOR PRE-K, TRANSITIONAL K, AND 4K

Children who enter kindergarten with high levels of math knowledge do better in math and in reading in later grades (Duncan, et al., 2007), so preparing Pre-K students to do well in kindergarten is crucial for later success and enriches children's daily lives.

Sources of the Foundational and Achievable Goals in the Pre-K Program

The **National Research Council's Committee on Early Childhood Mathematics** released a summary of research and recommendations to address the crucial national need to prepare all children to do well in kindergarten. The research and recommendations in this report, called Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity (National Research Council, 2009), provide the major research basis for the *Math Expressions Pre-K Program*. The report identified major foundational and achievable goals and approaches to be used in early childhood. The goals focused on the Number, Relations, and Operations Core and on the Geometry, Spatial Thinking, and Measurement Core. The *Math Expressions Pre-K Program* implements these goals in its Number and Geometry Learning Sessions, Daily Routines, and Math Centers.

Some of these major recommendations are summarized and exemplified in Focus in Prekindergarten (2010), published by the **National Council of Teachers of Mathematics** and the **National Association for the Education of Young Children**. This is a teacher-friendly version of the goals. The Math Expressions Pre-K Program also implements the position statement on early childhood learning by the National Council of Teachers of Mathematics (2016). Thus, the Math Expressions Pre-K Program implements the research-based recommendations of these two major professional organizations. State Pre-K goals are generally consistent with the recommendations of these organizations and of the National Research Council Report and thus are supported in the *Math Expressions Pre-K Program*.

THE TEACHING/LEARNING COMPONENTS

A crucial recommendation of the National Research Council's Report is that children must have sustained focused teaching/learning experiences in mathematics. Just learning through free play activities is not sufficient. There is too much social-cultural knowledge and language that must be learned for children to learn solely from each other or solely from materials. For this reason children in the *Math Expressions Pre-K Program* experience two 30-minute **Learning Sessions** a week led by the teacher. These Learning Sessions can be done in whole class or smaller groupings, depending on the size of the class. Children also work together four times a week in 15-minute **Math Centers** that provide explorations of materials and ideas before a lesson or activities that follow a lesson and deepen the concepts. A third focused teaching/learning experience is **Daily Routines** that focus on learning the count word sequence and its relationship to number symbols and to quantities shown and felt in patterns, fingers, and actions. These routines are done with the whole class using a poster on the wall that shows numbers and patterned quantities in order. Children work in Part A on the number-word sequence from 1 to 10, in Part B on the number-word sequence from 1 to 20 and especially 11 to 20, and in Part C on the number-word sequence from 1 to 50 and from 50 to 100 and on the sequence of tens multiples as the foundation of counting to 100. Having a child lead the counts adds interest and challenges everyone to work together. **Tutorial Time** specifies 5-minute activities the teacher does to support less-advanced children's understanding. Additionally, children receive help from the teacher and other children during the lessons and from an adult and other children during math center time.

Children connect mathematical concepts in the lessons and math centers to the real world in pictures and in their classrooms. The **Math Throughout the Day** program feature helps children see and discuss mathematical objects in their classroom environment. Special aspects to ensure such relating are the Number Noticer, Number Hiders, and Snack Number of the Day features. Children also find and discuss numbers of objects in pictures, and they play number games involving animals, snacks, and markets.These special math "noticing" activities reflect recent research indicating that young children who spontaneously notice numbers of things in their environment are more advanced in their other number knowledge (Hannula, Lepola, & Lehtinen, 2010; Hannula-Sormunen, Lehtinen, & Räsänen, 2015) and that such spontaneous noticing can be enhanced through guided focusing activities that we include (Hannula & Lehtinen, 2005). Facilitating such spontaneous noticing for all children is empowering and can strengthen math ideas. **Home Connections** provide ongoing communication with children's families. Weekly letters explain what children are learning and provide opportunities for children to demonstrate what they have learned and to continue learning at home.

THE MATH TALK COMMUNITY

A second major recommendation of the National Research Council's report is that children need to see mathematical ideas with objects and pictures and they need to discuss their thinking. The *Math Expressions Pre-K Program* uses a **Math Talk Community** approach in which ideas are elicited from children, and the teacher clarifies and extends children's ideas and verbal expressions. The teacher also can model when needed, but the major focus is always on eliciting and extending what children can do and say. Thus there is a major and continuing focus on building children's language about mathematical concepts. The Teacher Guide gives sample child responses within each lesson to indicate things that children might say. Lessons also give **Math Talk in Action** examples of children giving different responses and interacting with their responses. The teacher may expand or extend some responses to provide opportunities to learn and use language. A vocabulary list is given on the overview page for each week. Each week also has activities for English learners that are related to the lesson.

MathTalk in Action

Lin, tell us about your partners of 4.

Lin: I have 3 red circles and 1 yellow circle to make 4. Erica has the same colors, but hers are in different places.

Yes. Raise your hand if you show partners 3 red and 1 yellow.

Pedro: I have partners 3 and 1, but I have 3 yellow and 1 red.

Who has different partners?

Kiran: I have 2 and 2.

Walter: I do too!

Cathy: Look! I had 3 yellow circles and 1 red circle like Pedro, but I can turn over 1 yellow circle, and now I have 2 red and 2 yellow like Kiran and Walter.

So, do we all have 4 circles altogether? Let's count to be sure. Ready? 1, 2, 3, 4 circles. We all have 4 circles. We have different partners that make 4, but everyone has 4 in all.

MathTalk in Action

Describe how your circles are arranged alike or differently.

Eli: Mine are in a row.

Brianna: My circles are in a tower.

Julie: My 2 red circles are here and my 2 yellow circles are here.

Yes, your 2 red circles are on your left, and your 2 yellow circles are on your right.

Dylan: My 2 red circles are above, and my 2 yellow circles are below.

Odette: My circles go red, yellow, red, yellow, but I still have 2 red and 2 yellow.

Nick: My circles go red, red, red, yellow. I have 3 red circles and then 1 yellow circle.

Mario: Mine show partners of 3 and 1, but my 1 yellow circle is above the 3 red circles.

Gemma: Mine show partners of 3 and 1, but my 1 yellow circle is first.

Fran: I made a tower with my 3 red circles. My yellow circle is on the side.

So, our circles are arranged differently, but we all have 4!

THE PUZZLED PENGUIN PUPPET

One special aspect of this learning community is the **Puzzled Penguin puppet**. During a learning session the Puzzled Penguin makes a specified error that is typical of errors that children make. Children then need to help Puzzled Penguin understand why the error is incorrect and then show and explain to Puzzled Penguin how to correct the error. Puzzled Penguin is used in all grades Kindergarten to Grade 6 in *Math Expressions* because research finds that it often is easier to find and explain an error than to carry out a full math task entirely correctly. Typical errors also can be identified and discussed in class in this helpful way. Finally, such activities support the nurturing helping environment crucial for a Math Talk Community. The puppet is used in Pre-K to enliven this feature for the young Pre-K children and make it more playful.

Puzzled Penguin

Use the Puzzled Penguin to check whether children recognize that a square is also a rectangle.

Hold up the big purple square.

- *Puzzled Penguin says that this shape is not a rectangle. Is Puzzled Penguin right or wrong? Why?* Puzzled Penguin is wrong; the shape is a square and a rectangle. Elicit as many reasons as possible.

Puzzled Penguin

Have Puzzled Penguin count a row of 9 squares, skipping over a middle square and forgetting to point to the square and say the number word.

- *Did Puzzled Penguin count correctly?* No, Puzzled Penguin, you skipped this square. You have to count every square. Try again. Be extra careful.

Have Puzzled Penguin count correctly and say: *Thank you! If I am careful, I can count correctly. I have to count each square.*

TEACHER GUIDE AND SUPPORT FOR TEACHING

The Teacher Guide provides guidance for everything a teacher needs to teach the program. Each week contains an overview page summarizing the contents of the program components for that week. Detailed discussions of the two learning sessions for the week follow. These contain teaching dialogue, Math Talk in Action, pictures of the activities, and summaries of the other components for the week: Math Throughout the Day, Teaching Notes, English Learners, Check Understanding, and Tutorial Time. The last page for each week describes the Math Center activities including the Advanced Learner activities.

Math Throughout the Day

Children continue these activities:

1. Count when lining up.
2. Choose the Snack Number of the Day.
3. The Number Noticer places number cards 1–5 for objects in the classroom, and Number Hiders put pictures of 1–5 things for the Number Noticer to find.

Compare Items Children also look for ways to compare 1 to 5 items. They can compare crayons, geometric shapes, pieces of fruit, chairs, and tables, as well as items at home, such as dishes, cups, paper towels, and napkins.

Repeating Patterns Children can also make repeating patterns in school and at home using blocks, toys, small pieces of colored paper, and drawings with crayons or markers.

Shapes Ask children to look for and describe 3-sided and 4-sided shapes they see in the classroom. As they do, encourage them to extend their descriptions using the correct vocabulary.

side
corner
square corner
triangle
square
rectangle

Week 13 Find Partners of 5 and Find More/Less

Math Centers

This week each child does one or two of these activities during each of the four 15-minute Math Center sessions. Everyone plays *The Pet Game* on at least two days.

The Pet Game

Have pairs of children play *The Pet Game*. Children will read the number they roll and move the game circle that number of spaces, saying the numbers on each space. Next, they take that many Inch Squares for their "pretend pet" and place them on the square above the game circle.

Materials:
- Gameboard 1: *The Pet Game*
- Inch Squares
- game circles
- number cube

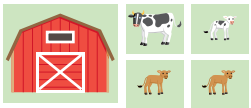
Animals Hiding in the Barn

Pairs of children find partners of 2 to 5.

Identifying Partners Partners agree on a number of animals, and put that many animals next to the barn. They take turns finding and naming partners for that number.

Hiding Partners The pairs agree on a number of animals, and put that many animals below the barn. One child closes and covers his or her eyes and says "Ready!" The other child then hides some animals under the picture of the barn and asks, "How many are hiding in the barn?" With eyes opened, the first child figures out how many are hiding by seeing how many are missing, by using fingers, or by knowing what number is the partner of the number of visible animals.

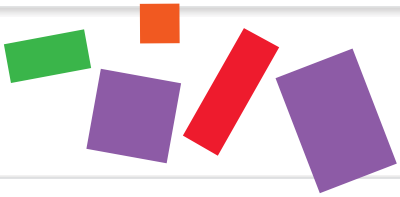
Materials:
- Number Story Cards: Animals
- Number Story Cards: Barns

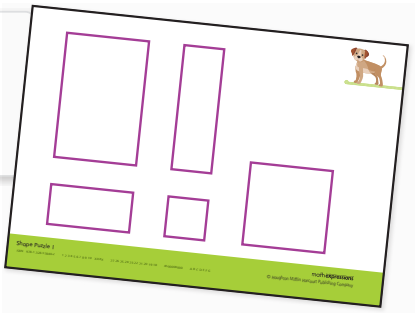


Shape Puzzles

Children use sets of five rectangles to practice solving the Dog/Cat and the Goldfish/Rabbit puzzles.

Materials:
- sets of five rectangles (1" by 1", 1" by 2", 1" by 3", 2" by 2", 2" by 3")
- Shape Puzzles 1 and 2





Advanced Learners

Comparing Mat

Children work in pairs. One child sets out two rows of Inch Squares on the comparing mat and the other child tells the comparison in two ways—using *more* and using *less*. Some children may set out equal numbers of squares and may use the terms *same, same as, equal,* or may say that, for example, "*Both numbers are 5.*"

Materials:
- Workmat 5: Compare to 5
- Inch Squares

The number and geometry materials are colorful and engaging. Manipulatives and some print materials are simple and clear so that children can focus on the numbers or shapes without distracting art. This is important because research on Pre-K materials has found that many counting books and other materials are too complex, distracting, and misleading to support quality learning (e.g., Ward et al., 2017).

Other program materials have realistic pictures and games in real-world settings to relate mathematical concepts to real-world situations. However, the design of these materials is always focused

on the quantities or shapes and distracting or irrelevant features are minimized. Many specially designed puzzles support children's spatial reasoning and composing and decomposing with shapes. Children play games that involve counting on a number path of written number symbols in a real-world setting. These games develop deeper understanding of the counting word list, of the numbers that follow each other, of the written number symbols, and of the quantities that the number words and number symbols represent. These games are discussed more fully below with respect to the research of Professor Robert Siegler.

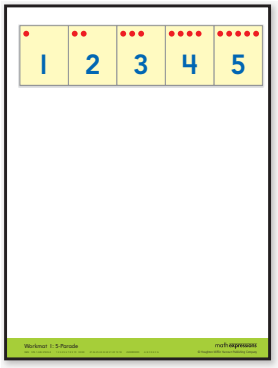
THE LEARNING PATHS IN NUMBER AND GEOMETRY

Major aspects of the National Research Council Report's **foundational and achievable goals in number** are

- to support perceptual and conceptual subitizing (recognizing how many without counting) of small groups,
- to achieve fluency in counting,
- to relate these two approaches to support understanding that the last counting word tells how many in a group, and
- to relate all of these aspects to written number symbols and spoken number words.

These goals require children to have many repeated related experiences to build the conceptual web of understandings. The early focus on cardinalities of small groups (how many) is crucial because these visual experiences provide meanings for number words and number symbols. Research has indicated that children whose mothers involve them in labeling the quantities of sets of objects show higher levels of mathematical achievement in preschool and first-grade math (Casey et al., 2016). The relations goals for numbers are to enable children to use perceptual, counting, and matching strategies to compare two numbers. The operation goals for numbers are for children to use conceptual subitizing and cardinal counting to solve simple situation, word, and oral number word problems.

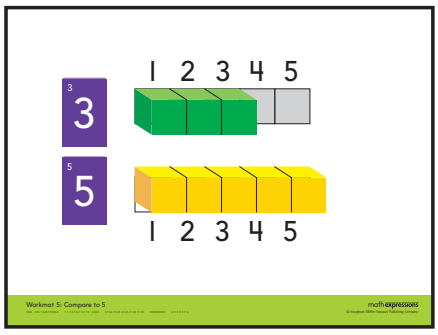
WORKMAT 1: 5-PARADE

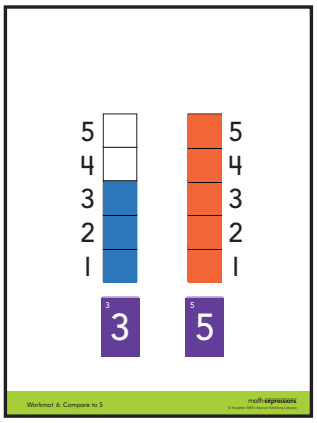


NUMBER TILES 1—4

TWO-COLOR COUNTERS OR **INCH SQUARES** OR **INCH CUBES**

Crucial for these early concepts is composing and decomposing numbers 5 and less to see two numbers hiding inside (the partners of a number). The number goals are prerequisites for the relations and operations goals, so the lessons in Number Part A focus on understanding the number goals for numbers 1 to 4. Part B extends the number work to 5, builds the relations (comparing) goals, and begins the experiences with operations. Part C extends children's experiences with relations (comparing) and operations. Parts B and C also include making, discussing, and generalizing repeating patterns, drawing from recent research by Professor Rittle-Johnson to be discussed in more detail below.



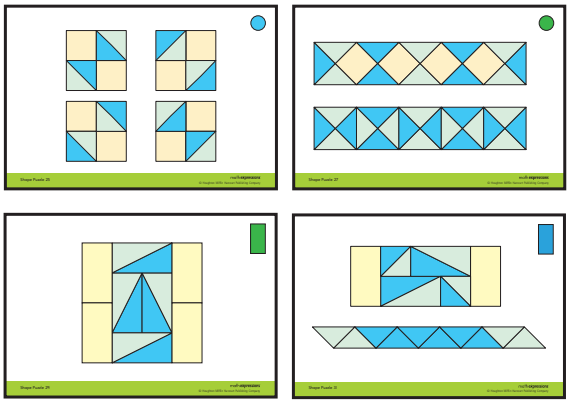


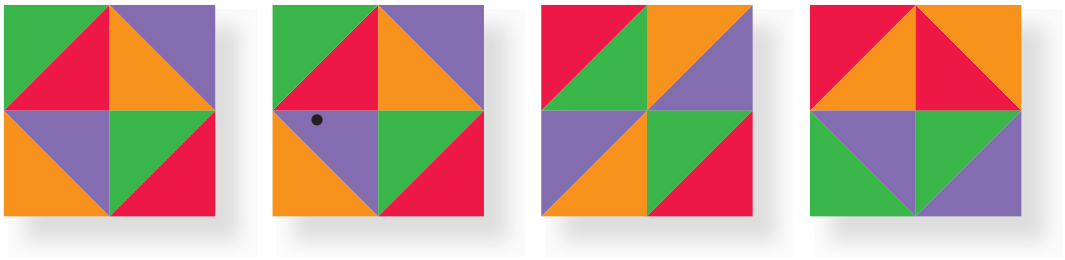
Many of the activities for the number goals are drawn from the activities with larger numbers in *Math Expressions Kindergarten*. These experiences work well with young children and have enabled high levels of learning. *The Math Expressions Pre-K Program* focuses for six weeks on the small numbers from 1 to 4 so that children who enter Pre-K not even being able to count can successfully build the complex web of knowledge with these small numbers. The program uses multi-level activities that enable children to engage at various levels of knowledge. So, some children in a learning group can focus on the early connections they need to make, while children who enter Pre-K having a considerable amount of number knowledge can participate and learn at a higher level. The Math Talk Community allows these more advanced children to see and point out relationships that other children may not yet see. The multiple levels of the activity allow children to deepen and extend and generalize their knowledge in ways that are seldom possible without the materials and activities in the program. There also are activities in the Math Centers for advanced learners that extend from the basic lessons. See the later discussion of the learning path of multi-level activities to see how activities enable all children to participate and learn productively.

Major aspects of the **foundational and achievable goals in Geometry, Spatial Thinking, and Measurement Core** are analyzing, discussing, composing, and decomposing shapes to support and develop various aspects of spatial thinking and basic experiences with measurement. An important recommendation of the National Research Council's report (2009) is that work with shapes must include substantial work with shapes that have right angles—rectangles (including squares) and triangles that have right angles (triangles that compose to make rectangles and squares). Many Pre-K programs include work with pattern blocks, but these are based on equilateral triangles as the composing unit and only a few special shapes can be composed from equilateral triangles. Such limited shapes also feed into the restrictive views many young children have of a triangle as only equilateral. Understanding right-angled shapes is crucial for much of later geometry, especially area measurement and the visual spatial structuring involved in such thinking (Clements et al., 2018). Special 2D shapes were developed for the *Math Expressions Pre-K Program* to support children to do

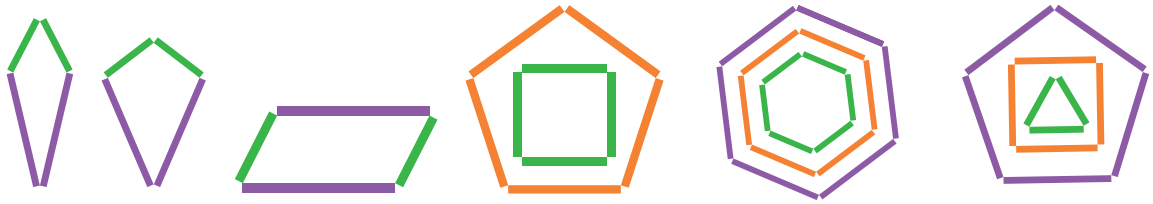
extensive work composing and decomposing and discussing relationships among these three-sided and four-sided shapes with right angles. Children then generalize their work with these shapes to many different shapes that have three and four sides. These goals are the focus of Geometry Part A.

Shape Puzzles 25–32 (1 set per pair)





In Geometry Part B children have introductory experiences with area and volume using the square inches and cubic inches they have been using in the number lessons. They explore, compose, and decompose 3D shapes and use relational spatial language to describe spatial relations between two shapes. They extend their earlier work composing right-angled shapes to more complex patterns and discuss patterns they see and make. In Geometry Part C, children explore length measurement and use length strips to make shapes with from three to ten sides and compose

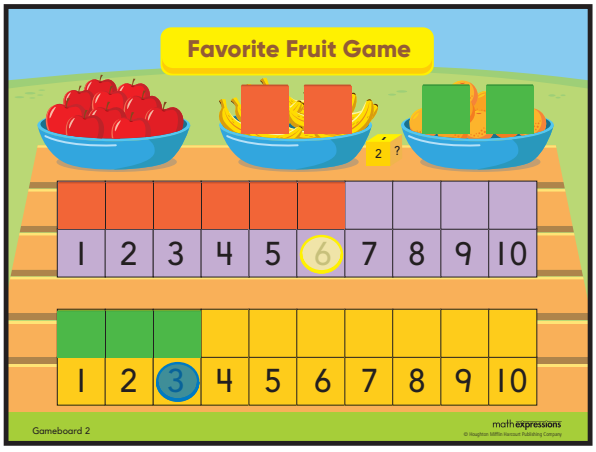


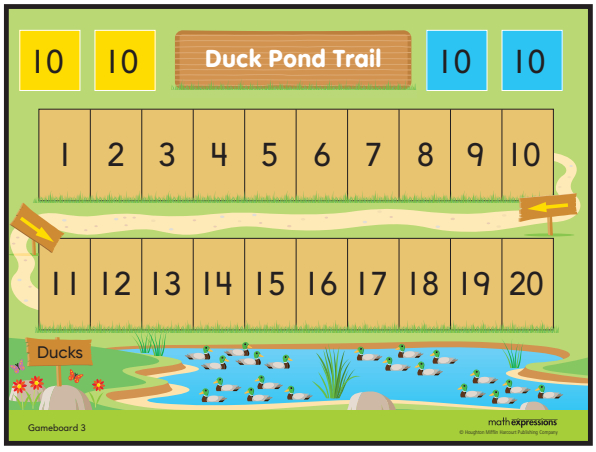
various shapes. Throughout this work on geometry and spatial thinking children put shapes on many puzzles and discuss how they do so, developing their shape and spatial awareness and geometric and spatial language. Work with these puzzles supports children through the research-based levels in composition of geometric shapes identified by Clements and Sarama (2014). Children begin with very simple puzzles so that everyone can experience success and proceed to more-advanced puzzles with and without outlines of the composing shapes.

NUMBER PATH COUNTING GAMES AND REPEATING PATTERNS

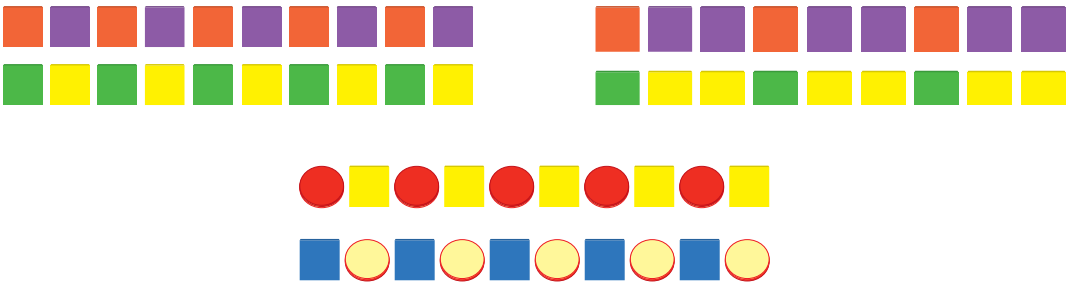
More recent research (e.g., Clements & Sarama, 2014) was also surveyed to ensure that the program includes the latest research-based learning activities. One important such topic is the number path games reported by Siegler and Ramani to increase numerical learning (e.g., Ramani & Siegler, 2008). Professor Robert Siegler, Carnegie-Melon University, is a prominent cognitive psychologist who studies math cognition. I deepened and extended his number path game for use in the *Math Expressions Pre-K program*. Siegler's game involves children counting along a number path (squares with numbers from 1 to 10 in order inside each square), much like other preschool games involving moving along numbered squares or circles. In his game children spin a spinner to find out how many they can move. His spinner had a "1" on one half and a "2" on the other half. The key improvement in Siegler's game compared to other preschool games is that children have to say the number on the squares on which they move their token rather than saying the number they had spun. So in Siegler's game, children who were on a 3 and spun a 2 would say, "4, 5" as they moved rather than "1, 2". This facilitates learning and automatizing the number word list auditorily and visually, learning the number that comes after a given number, and counting on 1 or 2.

I made several improvements and extensions to his game so that it could be productive over the whole year. Children begin with a cube that contains 0s and 1s on the sides so that they do not have to count on 2 at the beginning. They begin with a board from 1 to 4 to build knowledge of these small numbers. Their tokens are clear colored circles so that they can see the numbers onto which they are moving. Children also collect things as they move; this helps them connect the counting word list to cardinal numbers and see that the last counting word tells how many in all. Children place these items on squares adjacent to the squares on which they move, so as they have counted on the 1, 2, 3 numerals they have 3 items on the squares beside these numbers (see below in the Favorite Fruit Game). After children have played a few times, they predict before each roll what will happen (e.g., I am on 2. If I get a 1, I will move to 3.). They describe what happened after they move (I moved from 2 to 3), and take 1 or 0 objects, depending on which they rolled. These verbalizations help them reflect on their actions and strengthen the numerical connections among the written symbols, spoken number words, and quantities of objects. There are counting path games to 4, 10, 20, and 40. These games are in real-world contexts. Children collect pets in the Pet Game, collect fruits in the Favorite Fruit Game, and collect duck food pellets on the Duck Pond Trail game. In the Down, Up, Over to 40 game children have special short-cut or go-back moves. Children play first with a cube having 0s and 1s and then move to a cube with 1s and 2s. This cube also has a ? for which they choose 1 or 2. They need a 1 to finish the game if they are on the next-to-last square.





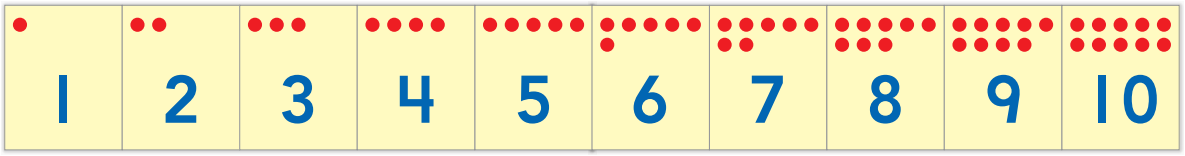
Another important new research-based topic in the *Math Expressions Pre-K Program* is a focus on **higher levels of repeating patterning**. Rittle-Johnson et al. (2015) identified levels of repeating patterns that children can demonstrate: *duplicate, extend, abstract, and explicitly recognize the smallest repeating unit of a pattern*. The higher two levels are considered particularly important for mathematical reasoning. These higher levels are not frequently implemented in a Pre-K program, although Rittle-Johnson considers them to be accessible and crucial for four-year-old children (http://blogs.edweek.org/edweek/early_years/2016/02/ask_a_scientist_what_should_preschool_math_look_like.html). *Abstract,* the third level, means to generalize the pattern across color or shape or other attributes, as in seeing that red, blue, red, blue, etc. has the same repeating pattern as yellow, green, yellow, green, etc. Making these abstractions is facilitated by the six colors of the 1-inch squares and 1-inch cubes children use in the *Math Expressions Pre-K Program* number lessons. Children almost immediately are abstracting across color because they have different colors of objects. They use colors to say their pattern and also use ordinal words (first, second, first, second, etc.). The oral repetitions in these chants abstract the repeating pattern further. Children also do Rittle-Johnson's fourth level: They *identify the smallest repeating unit* in their pattern (e.g., red, blue) and separate the smallest repeating units in the patterns so that these repeating units can easily be seen by everyone. Thus, instead of just seeing which level of repeating pattern children spontaneously produce initially, children are helped to move through and show all levels very early. This is possible because of the Math Talk Community in which the lessons occur so that help and explanation are available from other children as well as from the teacher.



MULTI-LEVEL TEACHING ACTIVITIES

Children who enter any preschool class often vary considerably in their mathematical knowledge. The *Math Expressions Pre-K Program* can be successful with the whole range of children because of its multi-level teaching activities and the Math Talk Community used in the Learning Sessions and Math Centers. In these multi-level teaching activities, children build connected webs of knowledge starting from what they know. Activities have choices and opportunities to discuss math thinking. Concepts are supported visually, and children can enter in various places and connect various parts of the web in their own time. This approach supports equity because all children can succeed, their background knowledge and thinking are supported and affirmed, and all children are supported to full competence with the Pre-K goals.

One such activity used throughout the year in the number Learning Sessions is the 5-Parade Workmat activity. The 5-Parade Workmat has a row of yellow rectangles along the top, each containing a numeral and that many dots. This 5-Parade is part of the large Number Parade poster from 1 to 10. This Number Parade is used throughout Part A in the Daily Routines. The poster shows the numbers from 6 to 10 using groups of 5 (e.g., 7 is 5 and 2). These groups match the 5 fingers children have on each hand. A Student Leader points to each number in order to lead the class



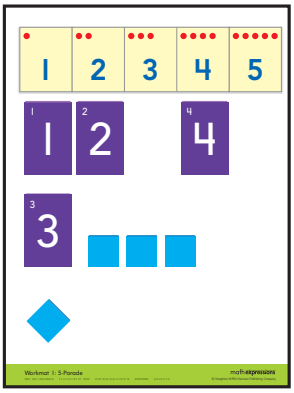
Number Parade

to count along the Number Parade, children show fingers in order for numbers, and they eventually identify numerals, dots, and fingers out of order. This poster is always up on the wall and visible so that children can use it during the 5-Parade Workmat activities.

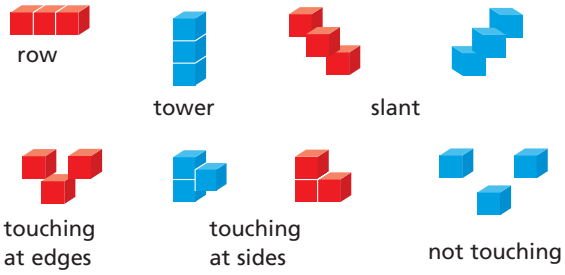
Children place purple foam tiles containing numbers on the 5-Parade Workmat. Each tile fits below a number. Children who have never seen numbers before can do such matching with a little experience. The teacher or a child then says a number, and all children pull that number down below. Children who do not know that number can watch and do what other children do, but rapidly they relate each visual symbol to its number word and display of dots because they count along the 5-Parade Workmat and the Number Parade every day. Children then use the objects for that week (2-colored circles, 6 colors of inch squares, or 6 colors of inch cubes) to show that number. Children do this in their own way and discuss different arrangements they make.

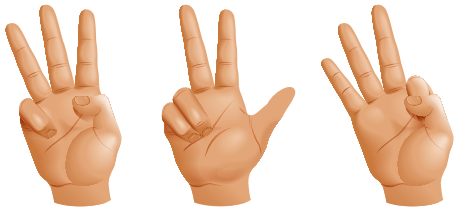
Children then show that number with their fingers. They do so in their own ways. Cultures around the world show numbers on fingers mainly in the three ways shown on the right. Children can also show part of a number on one hand and the rest on the other hand. Making, seeing, and discussing quantities in different ways enables children to generalize what a number means and provides visual support for adding and subtracting with small numbers.

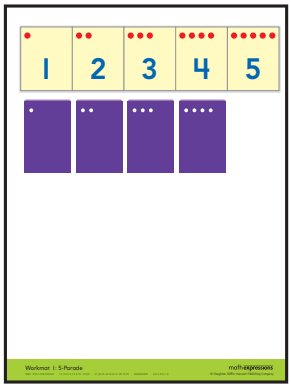
The backs of the purple foam tiles have the number of dots that matches the number on the front of each rectangle. Children eventually put these tiles in order by the number of dots. They can look at the dots above the numbers on the 5-Parade if they need extra support. At any time activities can be done without the 5-Parade Workmat to decrease the support available.



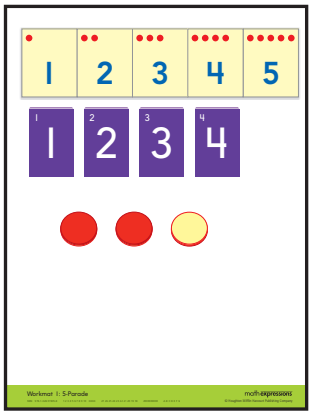
Possible Arrangements of 3 Cubes

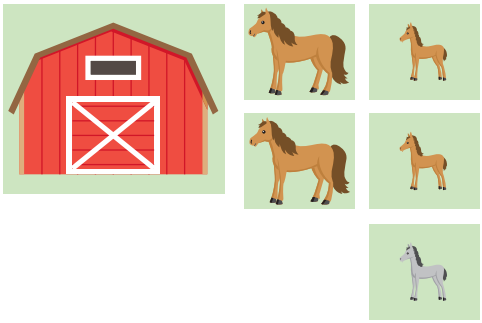




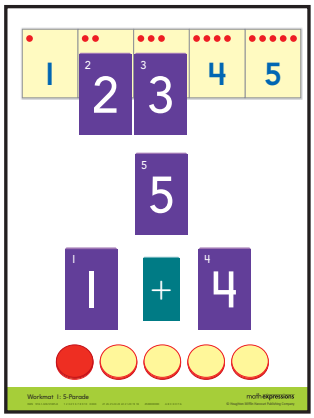


Children then begin to use objects with two colors to show the numbers hiding inside a given number, the partners into which a number can decompose. Below a child has shown 3 as 2 red and 1 yellow. Composing/decomposing a number into two quantities that make that number is foundational for much later numerical thinking including addition and subtraction. Children see and discuss different ways to show partners and then count and match to ensure that the different partners do total to make the same number. Children find partners of numbers on colorful posters and play games with animal cards and snack cards to show partners of a number. The cards always allow children to make and describe partners in their own way. For example, a child might say that 5 has partners 2 and 3 because there are 2 big horses and 3 little horses. Another child might say that 5 has partners 4 and 1 because there are 4 brown horses and 1 white horse.

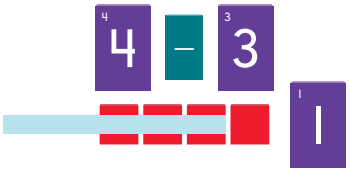




Finally, children record partners of a number using the purple tiles and a foam + sign. And eventually they place the total number they decomposed above the + sign. They tell adding stories made by these numbers using pictures and their imaginations. Finally, they show subtracting situations using a take-away strip to cover the number of objects taken away and a minus rectangle with the purple number rectangles. The take-away strip allows children to reflect on the numbers involved and see the total 4 decomposed into its partners 3 and 1. This will help children relate addition to subtraction as they have more experience.



One red flower is in the vase. Four yellow flowers are in a different vase. How many flowers in all?



There are 4 birds in the tree. Three fly away. How many birds are left in the tree?

Because children interact with objects and discuss their thinking in the Learning Sessions and in the Math Centers, teachers can continually assess what children know and help each child take the next steps. Tutorial time can be used for students who were ill or need a lot more experience, but many children can make connections for themselves in their own way. Children also help each other by modeling and explaining. The Teacher Guide contains an explicit Check Understanding prompt for each Learning Session. There is a number and a geometry Teacher Observation Checklist for each of the three Parts (A, B, C), and an "I Can …" page to send home at the end of each part so that parents can see what a child has learned.

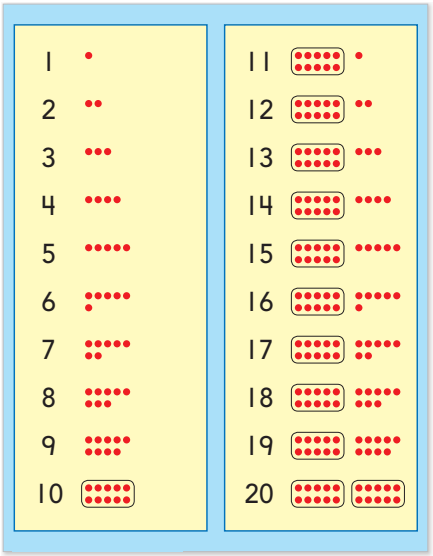
A FOCUS ON GENERALIZING CONCEPTS ACROSS COLORS, SHAPES, MATERIALS, SPATIAL ARRANGEMENTS, AND MATH DOMAINS

Half of the lessons in the *Math Expressions Pre-K Program* implement the number goals, and half implement the geometry goals, although most lessons involve both number and geometry ideas. Many of the number lessons involve geometric shapes (circles, 1-inch squares, and 1-inch cubes) because these shapes are visually simple and facilitate subitizing and counting. Children in many geometry lessons use number concepts as they subitize or count sides and corners of 3-sided and 4-sided shapes (and later, larger numbers of sides and corners). So children are continually building visual images and concepts in both domains and having opportunities to relate these.

As discussed above, children are supported through learning paths for various math domains, and they continually are helped to generalize concepts across various attributes. Children generalize concepts across colors, shapes, materials, spatial arrangements, different real-world situations, and different words.

DAILY ROUTINES AND LARGER NUMBERS

The Number Parade poster to 10 used in the Daily Routines in Part A is shown above. Part B develops children's understanding of and the counting words for numbers 1 to 20, and Part C does this for the counting word sequence to 100. The posters used for these Daily Routines are shown below. Children discuss patterns they see in each poster, relate spoken number words to the number symbols on the poster, and show fingers for numbers.



1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

MY OWN RESEARCH AND CONTINUATIONS OF THE PRE-K LEARNING PATHS

Many of my own studies over the past 35 years have contributed to our knowledge of how children learn mathematical ideas. These were included in the research base that was reviewed and summarized in the National Research Council Report (2009). Many of these studies are available on my website karenfusonmath.com (choose Publications in the menu). Summaries of teaching/learning approaches in *Math Expressions Kindergarten* are also on my website under Teaching Progressions, and videos of key activities in kindergarten can be found in Classroom Videos. These show some of the kindergarten activities for which children are preparing in the *Math Expressions Pre-K Program*.

- Dr. Karen Fuson

REFERENCES

Casey, B. M., Lombardi, C. M., Thomson, D., Nguyen, H. N., Paz, M., Theriault, C. A., & Dearing, E. (2016). Maternal support of children's early numerical concept learning predicts preschool and first-grade math achievement. *Child Development,* 89 (January 1), 156-173. https://doi.org/10.1111/cdev.12676

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach,* (2nd ed.). New York, NY: Routledge.

Clements, D. H., Sarama, J.. Van Dine, D. W., Barrett, J. E., Cullen, C. J., Hudyma, A., Dolgin, R., Cullen, A., & Eames, C. L. (2018). Evaluation of three interventions teaching area measurement as spatial structuring to young children. Journal of Mathematical Behavior, 50, 23-41. https://doi.org/10.1016/j.jmathb.2017.12.004

Duncan, G.J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A.C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., &Duckworth, K. (2007). School readiness and later achievement. *Developmental Psychology,* 43, 1428-1446.

Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction,* 15, 237–256.

Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of Experimental Child Psychology,* 107, 394–406.

Hannula-Sormunen, M. M., Lehtinen, E., & Räsänen, P. (2015). Children's preschool subitizing, spontaneous focusing on numerosity and counting skills as predictors of mathematical performance 6–7 years later at school. *Mathematical Thinking and Learning*, 17, 155–177.

National Council of Teachers of Mathematics and the National Association for the Education of Young Children (2010). *Focus in Prekindergarten*. Reston, Va.: National Council of Teachers of Mathematics and the National Association for the Education of Young Children.

National Research Council (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* Committee on Early Childhood Mathematics, C. Cross, T. Woods, & H. Schweingruber (Eds.). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development,* 79, 375-394.

Rittle-Johnson, B, Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: Adding patterns to the equation. *Early Childhood Research Quarterly,* 31, 101–112 And see Rittle-Johnson's discussion in http://blogs.edweek.org/edweek/early_years/2016/02/ask_a_scientist_what_should_preschool_math_look_like.html

Ward, J. M., Mazzocco, M. M., Bock, A. M., & Prokes, N. A. (2017). Are content and structural features of counting books aligned with research on numeracy development? *Early Childhood Research Quarterly,* 39, 47–63. https://doi.org/10.1016/j.ecresq.2016.10.002

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Pre-K Research Background and Design

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