HMH Into AGA Algebra 2

Unit 1: Functions and Equations

Unit 1 Project: STEM Task: Medical Anthropologist – Infection Detection
Unit 1 Learning Mindset Focus: Strategic Help-Seeking: Identifies Sources of Help

Module 1: Analyzing Functions

Recommended Pacing: 10 days

Module 1 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • understood that a function is a rule that assigns to each input exactly one output. • identified and interpreted key features of a function. • described the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. • identified and interpreted key features of function models represented in different ways.	Students: • determine domain, range, and end behavior of a function from its graph. • identify key characteristics of a function in context. • graph a combined transformation of a graph, including translations, a stretch or compression, and/or a reflection. • model with absolute value and quadratic functions. • investigate properties revealed by different function representations	Students: • will interpret parameters in terms of a context. • will transform and describe polynomial, rational, exponential, logarithmic, and trigonometric functions as being transformed from a parent function. • will graph key features of polynomial, rational, exponential, logarithmic, and trigonometric functions. • will compare properties of polynomial, rational, exponential, logarithmic, and trigonometric functions represented in different ways.

Module 1 Academic Vocabulary

absolute value of x	the distance between x and 0 on a number line; denoted as $\lvert x \rvert$
average rate of change	the ratio of the change in the function values, f (b) - f(a) , to the corresponding change in the x-values, b - a, over an interval [a, b]
end behavior of a function	a characteristic that describes what happens to the values of f(x) as the x-values increase or decrease without bound
parabola	a U-shaped curve that is the graph of g(x) = x 2
turning point	the point at which a function changes from increasing to decreasing or from decreasing to increasing
vertex	the turning point of a graph
even function	a function for which f(-x) = f(x) for all x in the domain of the function

odd function	a function for which f (-x) = -f(x) for all x in the domain of the function
parameter	a constant in a function rule that can be changed

Lesson 1.1 Domain, Range, and End Behavior – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can relate the domain, range, and end behavior of a function to its graph.

Learning Objective

Analyze, compare, and interpret functions across representations in the context of a real-world situation

Language Objective

Describe the domain, range, and end behavior of graphed functions using written words and appropriate notation forms.

Vocabulary

Review: end behavior, interval

Lesson Materials

graphing technology

Lesson 1.2 Characteristics of Functions and Graphs – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Mathematical Practices and Processes

- Attend to precision.
- Reason abstractly and quantitatively.

I Can Objective

I can relate the characteristics of real-world phenomena to characteristics of its function graph.

Learning Objective

Describe and interpret key characteristics of a function from its graph, and graph a function from a description of its characteristics.

Language Objective

Explain the key characteristics of a function graph based on a real-world situation.

Vocabulary

Review: average rate of change, decreasing, increasing, maximum, minimum, turning point, zero

Lesson 1.3 Transformations of Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can use a transformation rule to relate a preimage to its image

Learning Objective

Describe the effect on a graph given a transformation rule, and sketch the result of a transformation rule

Language Objective

Describe the effect on a graph given a transformation rule.

Vocabulary

Review: even function, odd function, parameter

Lesson 1.4 Transformations of Absolute Value and Quadratic Functions – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Mathematical Practices and Processes

- Attend to precision.
- Reason abstractly and quantitatively

I Can Objective

I can use parameters to identify changes in the key characteristics of a function.

Learning Objective
Identify and interpret the key characteristics of absolute value and quadratic functions.

Language Objective
Describe the similarities and differences in the characteristics of absolute value and quadratic parent functions.

Vocabulary
Review: absolute value, parabola, vertex

Lesson 1.5 Compare Functions Across Representations– 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified time interval. Estimate the rate of change from a graph.

Mathematical Practices and Processes

- Use appropriate tools strategically.

I Can Objective
I can compare the properties of two or more functions when they are represented in different ways.

Learning Objective
Compare the key features of individual functions represented in different ways.

Language Objective
Explain how to compare the minimum or maximum values of functions that are represented differently.

Module 2: Solve Quadratic Equations and Systems

Recommended Pacing: 8 days

Module 2 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used square root symbols to represent solutions of equations of the form x 2 = p. • performed operations with polynomials. • solved quadratic equations by factoring. • solved systems of linear equations by graphing, substitution, and elimination.	Students: • solve simple quadratic equations by taking square roots. • perform operations with complex numbers. • find complex solutions of quadratic equations by completing the square and using the Quadratic Formula. • Solve linear-quadratic systems by graphing, substitution, and elimination	Students: • will represent complex numbers on the coordinate plane. • will use complex numbers in polynomial identities and equations. • will find real and complex solutions to higher order polynomial equations. • will find solutions of the equation f(x) = g(x) , where f (x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Module 2 Academic Vocabulary

discriminant	the part of the Quadratic Formula inside the radical, b 2 - 4ac
completing the square	a process used to form a perfect square trinomial
Quadratic Formula	the formula x = -b ⍰} √ b— —— 2 - 4ac _________ 2a , which gives solutions, or roots, of equations in the form ax 2 + bx + c = 0, where a ≠ 0
complex number	any number that can be written in the form a + bi, where a and b are real numbers and i is the imaginary unit
imaginary number	the square root of a negative number
imaginary unit	the i component of an imaginary number in the form of ri, where r is a nonzero real number

Lesson 2.1 Solve Quadratic Equations by Taking Square Roots – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Solve quadratic equations by inspection, (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.

Learning Objective

Solve simple quadratic equations of the forms a x 2 + b = 0 and a x 2 = b, where a and b are nonzero integers.

I Can Objective

I can use equations to model and solve real-world problems.

Language Objective

Explain the steps needed to solve a simple quadratic equation by taking square roots.

Vocabulary

New: imaginary number, imaginary unit

Lesson 2.2 Operations with Complex Numbers – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Use the relation i 2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- Know there is a complex number i such that i 2 = -1, and every complex number has the form a + bi with a and b real.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can identify, add, subtract, and multiply complex numbers.

Learning Objective

Define and perform algebraic operations on complex numbers.

Language Objective

Explain how to identify real, imaginary, and complex numbers and use properties of equality to add, subtract, and multiply complex numbers.

Vocabulary

Review: imaginary number, real number
New: complex number

Lesson 2.3 Prove and Apply the Quadratic Formula – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Solve quadratic equations with real coefficients that have complex solutions.

Mathematical Practices and Processes

- Look for and express regularity with repeated reasoning.
- Attend to precision.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can find the solutions of any quadratic equation

Learning Objective

Prove the Quadratic Formula and then use it to solve quadratic equations.

Language Objective

Explain why the value of the discriminant can determine the number and type of solutions for a quadratic equation.

Vocabulary

Review: completing the square,
discriminant,
Quadratic Formula

Lesson 2.4 Solve and Graph Nonlinear Systems – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- Explain why the x-coordinates of the points where the graphs of the equations y = f (x) and y = g(x) intersect are the solutions of the equation f(x) = g(x) ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/ or g (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision

I Can Objective

I can use different methods to solve and graph nonlinear systems

Learning Objective

Solve and graph nonlinear systems and use nonlinear systems to model and solve real-world problems.

Language Objective

Explain methods used to solve and graph nonlinear systems.

Lesson Materials

graphing device

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Unit 2: Polynomial Functions and Equations

Unit 2 Project: STEM Task: Telecommunications Engineer – Network Functions
Unit 2 Learning Mindset Focus: Resilience – Monitors Emotions

Module 3: Polynomial Functions

Recommended Pacing: 4 days

Module 3 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • investigated the graphs of the parent quadratic and absolute value functions. • graphed transformations of the quadratic and absolute value functions. • wrote equations for and modeled with quadratic and absolute value functions. • sketched graphs of quadratic and absolute value functions in standard form.	Students: • investigate the graphs of the parent cubic function and the parent quartic function. • graph transformations of polynomial functions. • write equations for and model with transformations of polynomial functions. • sketch graphs of polynomial functions in intercept form.	Students: • will create polynomial equations and interpret their solutions as viable or nonviable. • will learn the Fundamental Theorem of Algebra. • will solve polynomial equations by finding zeros. • will solve real-world problems by graphing polynomial functions.

Module 3 Academic Vocabulary

cubic function	a type of polynomial function that has the standard form $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers and $a \neq 0$
local (or relative) maximum or minimum	the value of a function at a turning point that is greater than or less than all other domain values within a given interval
turning point	a point on the graph of a function where the function changes from increasing to decreasing or from decreasing to increasing
absolute (or global) maximum or minimum	the value of a function that is greater than or less than all other function values
polynomial function of degree n	has the standard form $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ where $a_n, a_{n-1}, \ldots, a_2, a_1$, and a_0 are real numbers and $a_n \neq 0$

Lesson 3.1 Graph Polynomial Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can use the degree of a polynomial function to determine the shape and characteristics of its graph.

Learning Objective

Graph polynomial parent functions and their transformations, identify key characteristics of the graph, and use a polynomial function to model a real-world situation.

Language Objective

Explain how to transform a parent function for even and odd polynomial functions.

Vocabulary

Review: cubic function
New: polynomial function of degree n

Lesson Materials

graphing calculator

Lesson 3.2 Analyze Graphs of Polynomial Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics

I Can Objective

I can use intercept form to graph and analyze polynomial functions

Learning Objective

Describe and interpret key characteristics of a polynomial function from its intercept form

Language Objective

Explain the steps needed to graph a polynomial function using the intercept form of the function.

Vocabulary

Review: local (or relative) maximum or minimum, turning point
New: absolute (or global) maximum or minimum

Lesson Materials
graphing calculator

Houghton Mifflin Harcourt.
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Module 4: Function Operations and Polynomials

Recommended Pacing: 10 days

Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • added, subtracted, and multiplied algebraic expressions. • used special products of binomials to model real-world situations. • factored trinomials using special factoring patterns.	Students: • add, subtract, and multiply functions. • add, subtract, and multiply polynomials. • factor polynomials. • divide polynomials.	Students: • will combine multiple function types using arithmetic operations. • will build new functions from existing functions using composition. • will use synthetic division to test the possible rational zeros of a polynomial function.

Module 4 Academic Vocabulary

binomial	A polynomial with two terms
monomial	a number or a product of a number and variables with whole number exponents
polynomial	a monomial or sum of monomials
trinomial	a polynomial with three terms
closure	a set of numbers has closure under a given operation if the result of the operation on any two numbers in the set is also in the set
degree of a monomial	the sum of the exponents of the variables in the monomial
degree of a polynomial	the greatest degree of monomial terms of the polynomial
irreducible factor	a factor of degree 2 or greater that cannot be factored further
leading coefficient	the coefficient of the first term of a polynomial written in standard form
polynomial identity	an equation stating that two polynomials are equivalent
synthetic substitution	a technique that uses an array to model the sequence of multiplications and additions needed to find the value of a polynomial function p(x) for any value of x

Lesson 4.1 Function Operations – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Combine standard function types using arithmetic operations.
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context

Mathematical Practices and Processes

- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.
- Model with mathematics.

Learning Objective

Apply basic operations (addition, subtraction, multiplication) to (generic) functions (defined by table or graph) and interpret in context.

I Can Objective

I can create a new function by adding, subtracting, multiplying, or dividing two existing functions.

Language Objective Explain the steps needed to combine two functions using the four basic operations to create a new composite function.

Lesson 4.2 Add and Subtract Polynomials – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Combine standard function types using arithmetic operations.
- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Model with mathematics

I Can Objective

I can add and subtract polynomial expressions, including those representing real-world situations

Learning Objective

Add and subtract polynomials and explain their relationship to the system of integers under these operations.

Language Objective

Explain the process of adding and subtracting polynomials and describe situations that are specific to addition or subtraction.

Vocabulary

New: degree of a monomial, degree of a polynomial, leading coefficient
Review: monomial, polynomial

Lesson 4.3 Multiply Polynomials – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add subtract, and multiply polynomials.
- Prove polynomial identities and use them to describe numerical relationships.
- Combine standard function types using arithmetic operations.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.
- Attend to precision.
- Reason abstractly and quantitatively

I Can Objective

I can multiply polynomials and use products of polynomials to model real-world situations.

Learning Objective

Multiply polynomials and verify and use polynomial identities

Language Objective

Explain the steps needed to multiply polynomials.

Vocabulary

New: polynomial identity
Review: binomial, trinomial

Lesson Materials

word description graphic organizer (Teacher Resource Masters), Open MiddleTM Worksheet (Teacher Resource Masters)

Lesson 4.4 Factor Polynomials – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Use the structure of an expression to identify ways to rewrite it.
- Prove polynomial identities and use them to describe numerical relationships.
- Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can use factoring to write a polynomial as the product of polynomials of lesser degree.

Learning Objective

Decompose polynomials into irreducible factors and identify zeros.

Language Objective

Discuss strategies for using special factoring patterns, explain the steps needed to implement factoring by grouping, and describe how to model and solve real-world situations with higher-degree polynomials.

Vocabulary
New: irreducible factor

Lesson 4.5 Divide Polynomials – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Know and apply the Remainder Theorem: For a polynomial p (x) and a number a, the remainder on division by x – a is p (a) , so p (a) = 0 if and only if (x – a) is a factor of p(x) .
- Rewrite simple rational expressions in different forms; write a(x) /b(x) in the form q (x) + r(x) /b(x) , where a(x) , b(x) , q(x) ,and r(x) are polynomials with the degree of r (x) less than the degree of b(x) , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mathematical Practices and Processes

- Attend to precision.
- Look for and express regularity in repeated reasoning.
- Construct viable arguments and critique the reasoning of others

I Can Objective
I can divide polynomials using both long division and synthetic division.

Learning Objective
Divide polynomials and explain their relation to the system of integers under this operation.

Language Objective
Explain why polynomials with real coefficients are closed under the operations of addition and multiplication.

Vocabulary
New: closure, synthetic substitution

Module 5: Polynomial Equations

Recommended Pacing: 4 days

Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • factored polynomials to find roots and solutions to equations. • used variables and expressions to represent quantities in a real-world problem. • understood the connection between the root of an equation and its graph.	Students: • understand and apply the Rational Zero Theorem and the Rational Root Theorem. • estimate the irrational zeros of polynomial functions. • understand and apply the Fundamental Theorem of Algebra. • write the equation of a polynomial function given its zeros. • solve real-world problems by graphing polynomial models.	Students: • will use polynomial zeros to graph rational functions. • will solve rational equations.

Module 5 Academic Vocabulary

complex conjugates	when a polynomial equation has complex roots, they come in pairs that can be written generally as $a + bi$ and $a - bi$
irrational conjugates	when a polynomial equation has irrational roots, they come in pairs that can be written generally as $a + b\sqrt{c}$ and $a - b\sqrt{c}$
zero of a function	any value of x for which $f(x) = 0$
multiplicity of a zero for a polynomial function	the number of times the zero is a factor in the factorization of the function
root of a polynomial equation	a solution of the polynomial equation $f(x) = 0$
successive approximations	closer and closer estimates for an irrational zero of a polynomial function found by systematically decreasing the interval that contains the zero until a desired place value is achieved

Lesson 5.1 Solve Polynomial Equations – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Know and apply the Remainder Theorem: For a polynomial (x) and a number a, the remainder on division by x - a is p (a) , so p(a) = 0 if and only if (x - a) is a factor of p(x) .
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Mathematical Practices and Processes

- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.
- Model with mathematics.
- Use appropriate tools strategically.

I Can Objective

I can use the Rational Roots Theorem to determine the roots of polynomials and to find solutions to polynomial equations.

Learning Objective

Use the Rational Roots Theorem to determine the real roots of polynomial equations and use the Rational Zero Theorem to find the real zeros of polynomial functions.

Language Objective

Describe the properties of a rational number, explain the difference between the root of a polynomial equation and the zero of a polynomial function, and explain the concept of closure with respect to polynomials.

Vocabulary

New: root, successive approximations

Review: zero of a function

Lesson 5.2 The Fundamental Theorem of Algebra – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p (a) , so p(a) = 0 if and only if (x - a) is a factor of p(x) .
- Graph polynomial functions, identifying the zeros of polynomials when suitable factorizations are available, and showing end behavior.

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can find the complex roots of polynomials and complex solutions to polynomial equations.

Learning Objective

Find all the zeros of a polynomial function using the Fundamental Theorem of Algebra, and write a polynomial function with the least degree possible given the function's zeros.

Language Objective
Explain the difference between complex conjugates and irrational conjugates.

Vocabulary
Review: complex conjugates, irrational conjugates
New: multiplicity

Lesson Materials
graphing calculator

HMH Into AGA Algebra 2

Unit 3: Rational Exponents and Radical Functions

Unit 3 Project: STEM Task: Astronomer – Radical Movements
Unit 3 Learning Mindset Focus: Perseverance – Learns Effectively

Module 6: Rational Exponents and Radical Operations

Recommended Pacing: 4 days

Module 6 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used variables and expressions to represent situations. • used properties of integer exponents to solve problems. • extended properties of integer exponents to define rational exponents.	Students: • define rational exponents in terms of roots. • translate between rational exponents and radical expressions. • simplify expressions involving rational exponents. • simplify radical expressions involving nth roots. • model with rational exponents.	Students: • will use properties of rational exponents to graph radical functions. • will solve equations involving radicals and rational exponents.

Module 6 Academic Vocabulary

index	the variable n in the radical expression $\sqrt[n]{a}$
nth root	when $b^n = a$, b is an nth root of a and is written as $b = \sqrt[n]{a}$
radical	any expression containing a radical ($\sqrt{\ }$) symbol

Lesson 6.1 Rational Exponents and *n*th Roots– 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can define rational exponents and nth roots and use them to solve real-world problems

Learning Objective

Define nth roots and rational exponents in two equivalent ways, evaluate rational exponents efficiently, and use rational exponents to solve real-world problems.

Language Objective

Explain both ways of representing a rational exponent in terms of a radical, and explain when a negative real number raised to a rational power is a real number.

Vocabulary

New: index, nth root, radical

Lesson Materials

scientific or graphing calculator

Lesson 6.2 Properties of Rational Exponents and Radical – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Rewrite expressions involving radicals and rational exponents using the properties of exponents

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics

I Can Objective

I can simplify and rewrite expressions containing rational exponents and nth roots.

Learning Objective

Extend properties of integer exponents to rational exponents and simplify and rewrite expressions containing rational exponents and nth roots.

Language Objective

Explain how properties of rational exponents can be used to simplify expressions containing radicals involving nth roots.

Module 7: Radical Functions and Equations

Recommended Pacing: 10 days

Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used function notation to evaluate a function for inputs in its domain. • determined the domain and range of a function in order to analyze and graph the function. • solved quadratic equations by taking square roots. • graphed quadratic and cubic functions.	Students: • compose functions. • find inverse functions. • recognize that the inverse of a quadratic function (with a restricted domain) is a square root function and that the inverse of a cubic function is a cube root function. • graph transformations of square root functions and cube root functions. • write square root functions and cube root functions given a graph. • model real-world problems with square root and cube root functions. • solve radical equations.	Students: • will understand the inverse relationship between exponential and logarithmic functions. • will solve exponential, logarithmic, and rational equations.

Module 7 Academic Vocabulary

cube root function	a function that takes any number n as the input and returns the positive number z which would have to be cubed to obtain n
inverse function	the function that “undoes” the operations of a function f(x) , denoted as f -1 (x)
square root function	a function that takes any positive number n as the input and returns the positive number z which would have to be squared to obtain n
composition of two functions	an operation that combines two functions by using the output of one function as the input of the other function
radical equation	an equation that contains a variable within a radical

Lesson 7.1 Inverse Functions and Function Composition – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Find inverse functions.
- Verify by composition that one function is the inverse of another.
- Compose functions.
- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Mathematical Practices and Processes

- Attend to precision.
- Reason abstractly and quantitatively.
- Look for and make use of structure.

Learning Objective

Students will find and verify inverse functions and use composition of functions to create new functions.

I Can Objective

I can find the inverse of a function and use composition of functions to verify inverse functions

Language Objective

Given a function, students should be able to explain how to find an inverse function both algebraically and graphically.

Vocabulary

Review: inverse function New: composition of functions

Lesson 7.2 Inverses of Quadratic and Cubic Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
- Verify by composition that one function is the inverse of another.
- Read values of an inverse function from a graph or a table, given that the function has an inverse.
- Produce an invertible function from a non-invertible function by restricting the domain.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can find the inverses of quadratic and cubic functions

Learning Objective

Students will identify and find inverses of quadratic and cubic functions and use such functions to solve real-world problems.

Language Objective

Students will be able to use and understand the terms square root function, quadratic function, cubic function, and parent function in the context of inverse functions.

Vocabulary
Review: cube root function, parent cube root function, parent square root function, square root function

Lesson Materials
index cards with functions

Lesson 7.3 Graph Square Roots Functions – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Graph square root, cube root, and piecewise-defined function, including step functions and absolute value functions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective
I can graph square root functions.

Learning Objective
Describe key features of the graphs of square root functions, graph square root functions, and determine a square root function from its graph.

Language Objective
Explain how to determine the transformations involved in graphing a square root function.

Lesson 7.4 Graph Cube Root Functions – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Graph square root, cube root, and piecewise-defined function, including step functions and absolute value functions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective
I can graph cube root functions.

Learning Objective
Describe key features of the graphs of cube root functions, graph cube root functions, and determine a cube root function from its graph.

Language Objective

Explain the steps involved in finding the transformations involved in graphing a cube root function.

7.5 Lesson Solve Radical Equations – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- Explain why the x-coordinates of the points where the graphs of the equations y = f (x) and y = g(x) intersect are the solutions of the equation f(x) = g(x) ; find the solutions approximately, e.g., using technology to graph the functions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Model with mathematics

I Can Objective

I can solve radical equations, including those with the variable on both sides.

Learning Objective

Solve radical equations involving square roots and cube roots, analyze graphs of radical equations, and use radical equations to model real-world situations.

Language Objective

Explain the steps needed to solve a radical equation.

Vocabulary

New: radical equation

Lesson Materials

graphing calculator

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Unit 4: Exponential and Logarithmic Functions and Equations

Unit 4 Project: STEM Task: Meteorologist – Predictions Under Pressure
Unit 4 Learning Mindset Focus: Challenge-Seeking: Sets Achievable Stretch Goals

Module 8: Exponential Functions

Recommended Pacing: 6 days

Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • analyzed linear functions and their graphs. • graphed transformations of quadratic, cubic, and absolute value functions. • created a quadratic, cubic, or absolute value function from a graph. • modeled real-world situations with linear and nonlinear functions. • calculated simple interest.	Students: • analyze exponential growth and decay. • graph transformations of exponential growth and decay functions. • create an exponential function from a graph. • model depreciation with an exponential decay graph. • graph and analyze the function $f(x) = e x$. • compare simple and compound interest. • model interest compounded over different periods of time.	Students: • will solve exponential equations, including those with base e. • will graph logarithmic functions, including those involving natural logarithms. • will use logarithms to model real-world problems.

Module 8 Academic Vocabulary

exponential decay	occurs when a population decreases at a consistent rate over time
exponential function	a function of the form $f(x) = b x$ where b is a positive constant other than 1 and the exponent x is a variable
exponential growth	an increase in number or size at a constantly growing rate
exponential growth or decay function	a function of the form $f(t) = a(1 \pm r) t$ where $a > 0$ is the initial amount and r is a constant percent increase or decrease for each unit increase in time t
compound interest	interest that is paid on both the principal and on the accumulated interest
growth or decay factor	the base, $1 \pm r$, in an exponential growth or decay function of form $f(t) = a(1 \pm r) t$
growth or decay rate	the variable r in an exponential growth or decay function of form $f(t) = a(1 \pm r) t$

Lesson 8.1 Exponential Growth and Decay Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can identify the effect of transformations on exponential functions to model situations of growth and decay.

Learning Objective

Identify the effect of transformations on exponential functions to model situations of growth and decay.

Language Objective

Explain how modifications of a parent function effect the transformation of an exponential function.

Vocabulary

Review: exponential decay, exponential function, exponential
growth
New: decay factor, decay rate, growth factor, growth rate

Lesson Materials

graphing calculator, graph of parent exponential function

Lesson 8.2 The Natural Base *e* – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x) , f(kx) , and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.
- Model with mathematics

I Can Objective

I can graph transformations of exponential functions having base e and use the graphs to solve real-world problems

Learning Objective
Graph transformations of exponential functions having base e and use the graphs to solve real world problems

Language Objective
Explain transformations of the function f(x) = e x and connect functions having base e to the use of their graphs and how they pertain to solutions of real-world problems

Lesson 8.3 Compound Interest – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices and Processes

- Model with mathematics.

I Can Objective
I can model the value of an investment that earns compound interest

Learning Objective
Rewrite exponential functions to better model real-world situations, including situations involving interest.

Language Objective
Explain the steps needed to write a function to model the value of an investment that earns compound interest.

Vocabulary
New: compound interest

Lesson Materials
graphing calculator

Module 9: Logarithmic Functions

Recommended Pacing: 6 days

Module 9 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • found inverses of functions. • defined and graphed exponential functions	Students: • understand logarithmic functions as the inverses of exponential functions. • convert between exponential and logarithmic equations. • evaluate logarithmic functions. • create exponential and logarithmic models. • graph logarithmic functions and transformations of those functions.	Students: • will use the properties of logarithms to rewrite logarithmic expressions. • will solve logarithmic equations

Module 9 Academic Vocabulary

common logarithms	logarithms with a base of 10
logarithm	the exponent to which a base must be raised to produce a given number
logarithmic function	the inverse of an exponential function
natural logarithms	logarithms with a base of e

Lesson 9.1 Logarithms and Logarithmic Functions– 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can define and evaluate logarithms

Learning Objective

Students will create, graph, and evaluate logarithmic functions in mathematical and real-world contexts.

Language Objective

Given an exponential or logarithmic function, students should be able to explain the process of how to find the inverse function and how to graph both functions.

Vocabulary

New: common logarithms, logarithm, logarithmic function, natural logarithms

Lesson 9.2 Graph Logarithmic Functions – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Reason abstractly and quantitatively.

I Can Objective

I can graph logarithmic functions

Learning Objective
Use the inverse relationship of parent logarithmic functions with exponential functions to determine points on their graphs, graph transformations of parent logarithmic functions, describe properties of the graphs of logarithmic functions, and describe the effects of transformations on properties of the parent function such as domain and range.

Language Objective
Describe how we can use the graphs of exponential functions to graph logarithmic functions.

Lesson Materials
graphing calculator

Lesson 9.3 Create Exponential and Logarithmic Functions– 2 Days

Focus on:
Apply and Practice

Mathematics Standards
- Fit a function to the data; use functions fitted to data to solve problems in the context of data. Use given functions or choose a function suggested by the context. Emphasize exponential models.
- Write a function that describes a relationship between two quantities.
- Graph exponential and logarithmic functions showing intercepts and end behavior.

Mathematical Practices and Processes
- Model with mathematics.

I Can Objective
I can create logarithmic and exponential equations to represent relationships between quantities.

Learning Objective
Students create logarithmic and exponential equations to represent relationships between quantities.

Language Objective
Explain the steps needed to create an exponential model for a data set algebraically and graphically.

Lesson Materials
graphing calculator

Module 10: Exponential and Logarithmic Equations

Recommended Pacing: 6 days

Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • graphed exponential functions. • used properties to solve polynomial equations and radical equations. • defined logarithms and defined logarithmic functions as the inverses of exponential functions. • graphed logarithmic functions.	Students: • investigate and prove properties of logarithms. • evaluate logarithmic expressions. • use logarithmic and exponential models to solve real-world problems. • solve logarithmic and exponential equations	Students: • will solve more complicated types of equations involving logarithms. • will use logarithmic functions to model real world phenomena with attention to constraints

Module 10 Academic Vocabulary

logarithm	the exponent to which a base must be raised to produce a given number

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Lesson 10.1 Properties of Logarithms – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- Define appropriate quantities for the purpose of descriptive modeling.

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

I Can Objective

I can develop and apply the properties of logarithms to simplify expressions.

Learning Objective

Students will be able to apply the properties of logarithms.

Language Objective

Explain the steps needed to simplify a logarithmic expression.

Vocabulary

Review: logarithm

Lesson Materials

scientific and graphing calculators

Lesson 10.2 Solve Exponential Equations – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- For exponential models, express as a logarithm the solution to a b ct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- Explain why the x-coordinates of the points where the graphs of the equations y = f (x) and y = g (x) intersect are the solutions of the equation f(x) = g(x) ; find the solutions approximately.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can use logarithms to find missing values for exponential models.

Learning Objective

Students use logarithms to find missing values for exponential models.

Language Objective

Students can explain the process to use logarithms to find missing values for exponential models and relate them to real-world applications.

Lesson Materials

graphing technology

Lesson 10.3 Solve Logarithmic Equations – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/ or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can solve logarithmic equations.

Learning Objective

Solve logarithmic equations graphically with help from a graphing calculator and algebraically by using the properties of logarithms and use them to solve real-world problems.

Language Objective

Explain the steps necessary to solve a logarithmic equation both graphically and algebraically.

Lesson Materials

graphing calculator, index cards, word definition map (Teacher Resource Masters)

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Unit 5: Rational Functions and Equations

Unit 5 Project: Chemical Engineer – Epoxy Proxy
Unit 5 Learning Mindset Focus: Strategic Help-Seeking: Asks Questions

Module 11: Rational Functions

Recommended Pacing: 6 days

Module 11 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • studied linear proportional relationships. • composed two functions. • analyzed transformations of functions. • graphed polynomial, exponential, and logarithmic functions. • found the inverse of a function. • rewrote a trinomial as the product of two binomials. • used long division to divide two polynomials.	Students: • write an inverse variation equation. • check data for inverse variation. • model with inverse variation. • graph simple rational functions. • write simple rational functions from graphs. • model with simple rational functions • identify the vertical, horizontal, and slant asymptotes in the graphs of rational functions. • graph and model with more complicated rational functions.	Students: • will add and subtract rational expressions. • will multiply and divide rational expressions. • will solve rational equations

Module 11 Academic Vocabulary

constant of variation	the number that relates two variables that are inversely proportional to one another; in the equation y = _ a/x, a is the constant of variation
inverse variation	a mathematical relationship between two variables such that $y = a/x$, where $a \neq 0$
rational function	a function of the form $f(x) = p(x) / q(x)$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$
slant asymptotes	linear asymptotes of rational functions that are neither vertical nor horizontal

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Lesson 11.1 Inverse Variation – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can recognize when two quantities show inverse variation and write an equation to model the relationship between the quantities.

Learning Objective

Determine whether given data represent an inverse variation, write an inverse variation to model a situation, and use an inverse variation equation to determine data values that vary inversely.

Language Objective

Explain how to determine whether data represent an inverse variation and how to write an inverse variation equation that models the data.

Vocabulary

New: constant of variation, inverse variation

Lesson Materials

graphing calculator

Lesson 11.2 Graph Simple Rational Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Attend to precision.
- Model with mathematics

I Can Objective

I can graph the rational function $f(x) = 1/x$ and use the graph's key features to graph transformations of this function.

Learning Objective

To graph the parent rational function $f(x) = 1/x$ and use its key characteristics to graph rational functions that are transformations of the parent function.

Language Objective

Explain the transformation of the parent rational function $f(x) = 1/x$ into the general rational function.

Vocabulary Review: rational function

Lesson 11.3 Graph More Complicated Rational Functions– 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Look for and make use of structure.
- Attend to precision.

I Can Objective

I can identify key characteristics of more complicated rational functions and use these characteristics to graph the functions.

Learning Objective

Identify key characteristics of graphs of rational functions with linear or quadratic numerators and denominators, and sketch graphs using these key characteristics.

Language Objective

Explain how to determine whether the graph of a rational function has a hole or a vertical asymptote at a value and why this procedure works.

Vocabulary

New: slant asymptote

Lesson Materials

graphing calculator

Module 12: Rational Expressions and Equations

Recommended Pacing: 6 days

Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • graphed rational functions. • added, subtracted, multiplied, and divided polynomials. • factored polynomials. • showed that the set of polynomials is closed under the operations of addition, subtraction, and multiplication. • found rational solutions of • polynomial equations.	Students: • simplify rational functions. • add, subtract, multiply, and divide rational expressions and models. • investigate properties of closure for rational expressions. • solve rational equations	Students: • will define and evaluate the basic trigonometric functions.

Module 12 Academic Vocabulary

extraneous solutions	values that are identified as excluded values in a given equation, and are therefore not part of the solution

Lesson 12.1 Multiply and Divide Rational Expressions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- Use the structure of an expression to identify ways to rewrite it.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can multiply and divide rational expressions.

Learning Objective

Students will simplify rational expressions, list excluded values for rational expressions, multiply and divide rational expressions, and use rational expressions to model and solve real-world problems.

Language Objective

Explain the steps needed to multiply and divide rational expressions, including listing excluded values.

Lesson 12.2 Add and Subtract Rational Expressions– 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Mathematical Practices and Processes

- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

I Can Objective

I can add and subtract rational expressions.

Learning Objective

Students will add and subtract rational expressions, identifying excluded values when necessary.

Language Objective

Explain the steps need to add and subtract rational expressions and find the least common denominator of rational expressions.

Lesson 12.3 Solve Rational Equations – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- Create equations and inequalities in one variable and use them to solve problems.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Mathematical Practices and Processes

- Use appropriately tools strategically.
- Attend to precision.
- Model with mathematics.

Learning Objective

Students solve rational equations graphically and algebraically.

Language Objective

Explain the steps needed to solve a rational equation including finding the excluded values for the equation.

I Can Objective

I can solve rational equations graphically and algebraically.

Vocabulary

Review: extraneous solution

Lesson Materials

graphing calculator

HMH Into AGA Algebra 2

Unit 6: Sequences and Series

Unit 6 Project: Computer Programmer – Iteration Calculation
Unit 6 Learning Mindset Focus: Perseverance: Checks for Understanding

Module 13: Explicit Formulas For Sequences and Series

Recommended Pacing: 6 days

Module 13 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • wrote functions and graphed them, including functions with restricted domains. • worked with various representations of functions, including tables, graphs, equations, and verbal descriptions. • wrote and interpreted sequences as functions on the integers. • wrote and interpreted exponential functions.	Students: • define mathematical sequences and series. • create and use rules for sequences. • use a formula for the sum of a series. • determine a term in arithmetic and geometric sequences. • define formulas for and calculate the sums of finite arithmetic and geometric series. • model with arithmetic and geometric series.	Students: • will define recursive functions. • will apply recursive rules and sequences.

Module 13 Academic Vocabulary

arithmetic sequence	a sequence whose consecutive terms differ by the same nonzero number
arithmetic series	the sum of the terms of an arithmetic sequence
common difference	the constant nonzero number that consecutive terms of an arithmetic sequence differ by
common ratio	the constant ratio between successive terms is a geometric sequence
geometric sequence	a sequence in which the ratio of successive terms is a constant
geometric series	the sum of the terms of a geometric sequence
index of summation	a variable used to count the terms of a series from the lower limit of summation to the upper limit of summation
rule	the equation used to describe a sequence
sequence	an ordered list of numbers

series	the expression formed by adding the terms of a sequence
sigma notation	a notation that uses the symbol Σ followed by an expression that efficiently represents a series
term	a number in the ordered list that makes up a sequence

Lesson 13.1 Define Sequences and Series – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can write sequences and series, and I can find general terms.

Learning Objective

Write sequences and series and find general terms.

Language Objective

Explain the steps needed to write sequences and series to find general terms.

Vocabulary

New: finite sequence, finite series, index of summation, infinite sequence, infinite series, lower limit of summation, rule, sequence, series, sigma notation, term, upper limit of summation

Lesson Materials

spreadsheet software

Lesson 13.2 Arithmetic Sequences and Series – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Mathematical Practices and Processes

- Model with mathematics.
- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.

I Can Objective
I can write arithmetic sequences and series and use them to model real-world situations.

Learning Objective
Write arithmetic sequences, find the sum of finite series, and use them to model real-world situations.

Language Objective
Explain the steps needed to write an arithmetic sequence or series.

Vocabulary
New: arithmetic sequence, arithmetic series, common difference

Lesson 13.3 Geometric Sequences and Series – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others

I Can Objective
I can write geometric sequences and series and use them to model real-world situations.

Learning Objective
Determine specific terms in a geometric sequence using values for known terms and the common ratio; use formulas to calculate the sum of a finite geometric series; and use geometric series to solve real-world problems.

Language Objective
Explain a method for determining the value of the first term of a geometric sequence when two of the terms are known, but the first term is not given.

Vocabulary
New: common ratio, geometric sequence, geometric series

Module 14: Recursive Formulas for Sequences

Recommended Pacing: 4 days

Module 14 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • wrote and interpreted sequences as functions on the integers. • created and applied explicit formulas for arithmetic sequences and series.	Students: • write and apply recursive formulas for arithmetic sequences. • translate between formulas for arithmetic sequences. • write and apply recursive formulas for geometric sequences. • translate between formulas for geometric sequences.	Students: • will apply the idea of recursion to construct proofs by mathematical induction.

Module 14 Academic Vocabulary

explicit formula	a formula that gives the nth term, an, as a function of the term's position number n in the sequence
recursive formula	a formula that gives the beginning term or terms of a sequence and then a recursive equation that tells how the nth term, an, is related to one or more preceding terms

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Lesson 14.1 Recursive Formulas for Arithmetic Sequences – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Mathematical Practices and Processes

- Look for and express regularity in repeated reasoning.
- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can write a recursive formula for an arithmetic sequence and translate between explicit and recursive formulas for arithmetic sequences.

Learning Objective

Find values of sequences from recursive formulas, describe recursively defined sequences as discrete functions, write recursive formulas for arithmetic sequences, relate explicit and recursive formulas of arithmetic sequences, and translate between those types of formulas.

Language Objective

Compare and contrast the domains of an explicit formula and an implicit formula of a sequence.

Vocabulary

New: explicit formula, recursive formula

Lesson Materials

spreadsheet software

Lesson 14.2 Recursive Formulas for Geometric Sequences – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Mathematical Practices and Processes

- Look for and express regularity in repeated reasoning.
- Attend to precision.
- Look for and make use of structure.
- Model with mathematics

I Can Objective

I can write recursive formulas for geometric sequences and translate between explicit and recursive formulas for geometric sequences.

Learning Objective

Write recursive formulas for geometric sequences, translate between recursive and explicit formulas for geometric sequences, and apply recursive formulas and explicit formulas for geometric sequences in real-world situations.

Language Objective

Explain the steps needed to write a recursive formula for a geometric sequence.

Lesson Materials

spreadsheet software

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Unit 7: Trigonometric Functions and Identities

Unit 7 Project: Solar Engineer – Optimal Sun-Shine
Unit 7 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 15: Unit-Circle Definition of Trigonometric Functions

Recommended Pacing: 6 days

Module 15 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • defined radians in terms of arc lengths. • defined trigonometric functions in terms of the ratio of sides in a right triangle. • determine the ratio of sides in special right triangles. • proved the Pythagorean Theorem for right triangles and used it to solve problems.	Students: • convert between degree measure and radian measure. • solve real-world problems involving arc length. • use special right triangles in a unit circle. • explore basic trigonometric functions for special angles. • evaluate trigonometric functions given a point. • use trigonometric functions to solve real world problems. • prove the Pythagorean Identity. • find the other trigonometric functions given the value of sin θ, cos θ, or tan θ.	Students: • will graph trigonometric functions. • will prove the Law of Sines and the Law of Cosines and use them to solve problems. • will prove other trigonometric identities.

Module 15 Academic Vocabulary

angle of rotation	an angle formed by the starting and ending positions of a ray that rotates about its endpoint
coterminal angles	two angles of different measure whose terminal sides coincide
radian measure	the ratio of the arc of length s to the radius r of the circle on which the arc lies
unit circle	a circle centered at the origin with radius 1

Lesson 15.1 Angles of Rotation and Radian Measure – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can mathematically express the relationship between the unit circle and radian measure.

Learning Objective

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle and convert between degrees and radians.

Language Objective

Explain radian measure in terms of the unit circle.

Vocabulary

Review: radian

New: angle of rotation, coterminal angle, radian measurement

Lesson Materials

index cards

Lesson 15.2 Define and Evaluate Basic Trigonometric Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3 , π/ 4 and π /6 , and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π - x in terms of their values for x, where x is any real number.

Mathematical Practices and Processes

- Look for and make use of structure.
- Use appropriate tools strategically.
- Model with mathematics.

I Can Objective

I can use the unit circle to define the trigonometric functions for all real numbers.

Learning Objective

Use the unit circle to evaluate trigonometric functions for special right triangles.

Language Objective

Describe the process of evaluating a trigonometric function in all quadrants of the unit circle.

Lesson Materials

graphing calculator and coordinate planes (Teacher Resource Masters)

Lesson 15.3 Use a Pythagorean Identity – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Prove the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and use it to find $\sin \theta$, $\cos \theta$, or $\tan \theta$ given $\sin \theta$, $\cos \theta$, or $\tan \theta$ and the quadrant of the angle.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can use a given trigonometric function value to calculate the values of other trigonometric functions by means of a Pythagorean identity.

Learning Objective

Prove the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and use it to calculate trigonometric ratios.

Language Objective

Explain the process of using the Pythagorean identity to solve problems when given the value of one trigonometric function.

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Module 16: Graph Trigonometric Functions

Recommended Pacing: 8 days

Module 16 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • described major characteristics of a function. • graphed transformations of functions. • defined the basic trigonometric functions using the unit circle.	Students: • graph trigonometric functions. • write trigonometric functions from graphs. • model problems with trigonometric functions. • determine phase shifts of sine and cosine functions.	Students: • will model real-world phenomena using cosine and tangent functions. • will solve equations modeling real-world phenomena using trigonometric functions.

Module 16 Academic Vocabulary

amplitude	the distance that the "crest," where the function's maximum value occurs, rises above the midline or the distance that the "trough," where the function's minimum value occurs, falls below the midline of sine and cosine functions
frequency	the number of cycles completed in a given unit of time
midline	the point halfway between the maximum value and minimum value of sine and cosine functions
period	the length of a function's interval of repetition
periodic functions	functions that repeat their values over regular intervals on the horizontal axis; sine and cosine functions are periodic functions

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Lesson 16.1 Graph Sine and Cosine Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can identify the key features of the graphs of the sine and cosine functions.

Learning Objective

Graph sine and cosine functions and identify key features of those graphs to model and solve real-world problems.

Language Objective

Explain how to graph sine and cosine functions and identify key features of those graphs.

Vocabulary

New: amplitude, frequency, midline, period, periodic function

Lesson Materials

Coordinate plane for trigonometry (Teacher Resource Masters), index cards, Parent sine function (Teacher Resource Masters)

Lesson 16.2 Graph Tangent Functions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics

I Can Objective

I can identify the key features of the graph of a tangent function.

Learning Objective

Graph tangent functions, showing period, intercepts, and Asymptotes.

Language Objective

Describe the key feature of the graph of tangent functions and compare different tangent functions.

Lesson Materials

graphing technology, calculator

Lesson 16.3 Translations of Trigonometric Graphs – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Identify the effect on the graph of replacing f(x) by f(x) + k, k·f(x) , f(kx) , and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
- Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Mathematical Practices and Processes

- Look for and make sure of structure.
- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can identify how f(x + h) and f(x) + k will shift the graph of a trigonometric function f (x) for constants h and k.

Learning Objective

Students will be able to perform translations on trigonometric functions and their graphs.

Language Objective

Students will be able to describe the translation of a trigonometric function.

Lesson Materials

index cards, sine and cosine functions (Teacher Resource Masters)

Lesson 16.4 Model Periodic Phenomena with Trigonometric Functions – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- Write a function that describes a relationship between two quantities.

Mathematical Practices and Processes

- Look for and make use of structure.
- Use appropriate tools strategically.
- Model with mathematics.

I Can Objective

I can use trigonometric functions to model periodic phenomena and solve real world problems.

Lesson Materials: graphing calculator

Learning Objective

Calculate parameters for the general sine or cosine function to model a real-world situation, use technology to generate a sine regression model for a real-world situation, and use a trigonometric model to answer questions about a real-world situation.

Language Objective

Explain the steps needed to create sine functions by calculating parameters and by using technology.

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Unit 8: Probability

Unit 8 Project: Archaeologist – Dig Deep
Unit 8 Learning Mindset Focus: Perseverance: Getting Unstuck

Module 17: Probability of Compound Events

Recommended Pacing: 6 days

Module 17 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • defined theoretical probability in terms of sets and set operations. • constructed and interpreted two-way frequency tables.	Students: • investigate theoretical and experimental probability. • explore intersections, unions, and complements. • calculate probabilities. • investigate and compute probabilities from two-way tables. • explore probabilities involving mutually exclusive events and inclusive events.	Students: • will calculate marginal, joint, and conditional probabilities. • will classify events as dependent or independent.

Module 17 Academic Vocabulary

joint relative frequency	the ratio of the number of people or objects in a particular category to the total number of people or objects
marginal relative frequency	the sum of the joint relative frequencies in a row or a column
complement	the set of outcomes in the sample space that are not included in a particular event
event	a subset of outcomes from the sample space
experimental probability	when a probability experiment is conducted for a certain number of trials
intersection	the set of outcomes that are in both events under consideration
inclusive events	another phrase for overlapping events
mutually exclusive events	two events that cannot occur at the same time
overlapping events	two events that have one or more outcomes in common
sample space	the set of all possible outcomes of an experiment
theoretical probability	the ratio of the number of outcomes in a given event to the total number of outcomes in a sample space

union	the set of outcomes that are in one event or the other

Lesson 17.1 Theoretical and Experimental Probability – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision

I Can Objective

I can calculate the theoretical or experimental probability of an event.

Learning Objective

Find theoretical and experimental probabilities for real-world situations.

Language Objective

Explain the differences between theoretical and experimental probability.

Vocabulary

New: complement, event, experimental probability, intersection, outcome, probability experiment, sample space, theoretical probability, trial, union

Lesson 17.2 Two-Way Tables and Probability – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.

I Can Objective

I can construct two-way tables and use them to calculate probabilities.

Learning Objective

Construct and use two-way tables to calculate probabilities.

Language Objective

Explain the steps needed to construct two-way tables.

Vocabulary

Review: joint relative frequency, marginal relative frequency, two-way frequency table, two-way relative frequency table

Lesson 17.3 Mutually Exclusive and Inclusive Events – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Apply the Addition Rule, P (A or B) = P(A) + P(B) - P(A and B) , and interpret the answer in terms of the model.
- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Reason abstractly and quantitatively.

I Can Objective

I can use probabilities to identify events as either mutually exclusive or inclusive.

Learning Objective

Determine whether two events are mutually exclusive or inclusive, and use various methods to determine the probability of two events, including using the Addition Rule.

Language Objective

Explain how to determine whether two events are mutually exclusive or inclusive by examining the values in a two-way frequency table.

Vocabulary

New: inclusive events, mutually exclusive, overlapping events

Module 18: Probability and Decision Making

Recommended Pacing: 6 days

Module 18 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used two-way tables to compute probabilities. • contrasted theoretical and experimental probability. • determined whether events were mutually exclusive or inclusive.	Students: • find conditional probabilities given a two-way frequency table, a two-way relative frequency table, or a formula. • determine and show the independence of events. • find the probabilities of two independent or dependent events. • analyze decisions using probabilities. • derive Bayes' Theorem.	Students: • will calculate and use probability distributions. • will use probabilities to construct and interpret confidence intervals.

Module 18 Academic Vocabulary

conditional probability	the probability an event occurs given that another event has already occurred
dependent event	the occurrence of one event that does affect the probability of the other event
independent event	the occurrence of one event that will not affect the probability of the other event

Lesson 18.1 Conditional Probability – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Understand the conditional probability of A given B as P (A and B)/P (B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Mathematical Practices and Processes

- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Reason abstractly and quantitatively

I Can Objective

I can calculate conditional probabilities using two-way tables and formulas.

Learning Objective

Interpret a conditional probability in terms of an assumed event; recognize conditional probabilities in real-world scenarios; calculate conditional probabilities with two-way frequency and relative frequency tables and formulas, and recognize that these methods are equivalent.

Language Objective

Explain how to calculate P(A | B) from a two-way frequency table.

Vocabulary

New: conditional probability

Lesson 18.2 Dependent and Independent Events – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can determine if events are independent or dependent and calculate probabilities of the events accordingly.

Learning Objective
Determine if events are independent or dependent and calculate probabilities of the events accordingly.

Language Objective
Explain the differences between the probabilities of dependent and independent events.

Vocabulary
Review: dependent events, independent events

Lesson 18.3 Analyze Decisions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Analyze decisions and strategies using probability concepts. (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.

I Can Objective
I can use conditional probability to analyze real-world decisions.

Learning Objective
Determine conditional probabilities using multiple methods, such as Bayes' Theorem or tree-diagrams; analyze conditional probabilities to make effective decisions.

Language Objective
Explain the differences between using Bayes' Theorem to make decisions involving conditional probability, and using a tree diagram to make decisions involving conditional probability.

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Unit 9: Statistics

Unit 9 Project: Signal Processing Engineer – Significant Signals
Unit 9 Learning Mindset Focus: Challenge Seeking: Makes Plans to Meet Goals

Module 19: Data Distributions

Recommended Pacing: 8 days

Module 19 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • distinguished between experimental and theoretical probabilities. • calculated probabilities of independent and dependent events. • summarized categorical data with two-way frequency tables and calculated relative frequencies. • used the mean to describe numerical data. • described the shape, center, and spread of data.	Students: • develop a symmetric binomial probability distribution. • calculate and analyze probabilities from a distribution. • fit a normal curve to a histogram and find areas under a normal curve. • use the standard normal distribution. • explore and classify sampling methods. • make predictions from a random sample. • develop and use a distribution of sample means and sample proportions.	Students: • will use normal distributions to establish confidence intervals and margins of error. • will make inferences from experimental data using probability distributions. • will use data from a sample to estimate a population parameter.

Module 19 Academic Vocabulary

biased sample	does not fairly represent the population and so can produce statistics that can lead to inaccurate conclusions about population parameters
binomial experiment	a probability experiment of n identical independent trials, where there are only two possible outcomes, success or failure
empirical rule	the consistent relationship between the mean and its standard deviation for all normally distributed data sets, modeled by the area between certain intervals under a normal curve
normal curve	A smooth bell-shaped curve that represents situations where the mean is in the center of the data and the percentages decrease symmetrically on both sides
probability distribution	a data distribution that gives the probabilities of the values of a random variable, which can be represented by a histogram with the values of the random variable along the horizontal axis

sample	the study of only some of a population's members in order to gather data
standard normal distribution	a normal distribution with a mean of 0 and standard deviation of 1
z-score	the number of standard deviations a given data value is from the mean of the data set

Lesson 19.1 Probability Distributions – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can define and display probability distributions for discrete random variables to model real-world scenarios or probability experiments.

Learning Objective

Explain the steps needed to create a binomial probability distribution.

Language Objective

Explain the differences between theoretical and experimental probability.

Vocabulary

Review: binomial experiment, binomial probability distribution, Pascal's Triangle, probability distribution, random variable

Lesson 19.2 Normal Distributions – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Mathematical Practices and Processes

- Look for and make use of structure.
- Use appropriate tools strategically.

I Can Objective

I can find percentages of data and probabilities of events associated with normal distributions.

Learning Objective

Identify normal curves and normal distributions and solve standard deviation problems using the empirical rule or the z-score for a data value.

Language Objective

Explain how to use standard normal distribution table and the z-score for a data value to find the percentage of data below a given value.

Vocabulary
Review: mean, standard deviation, New: empirical rule, normal curve, normal distribution, standard normal distribution, z-score

Lesson 19.3 Data Gathering Techniques – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Mathematical Practices and Processes

- Attend to precision.
- Construct viable arguments and critique the reasoning of others.
- Reason abstractly and quantitatively.

I Can Objective
I can recognize the relationship among populations, samples, statistics, and parameters, and identify representative sampling methods.

Learning Objective
Recognize the relationship among populations, samples, statistics, and parameters, and identify representative sampling methods.

Language Objective
Identify and describe a sampling method.

Lesson Materials:
Word Description (Teacher Resource Masters), notecards

Vocabulary
Review: categorical data, numerical data
New: biased sample, census, parameter, population, proportion, representative sample, sampling, statistic

Lesson 19.4 Sampling Distributions – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Mathematical Practices and Processes

- Model with mathematics.
- Reason abstractly and quantitatively.

I Can Objective
I can describe how the mean of a sampling distribution, the corresponding population mean, and the population proportion are related.

Learning Objective
Use data from a sample survey to estimate a population mean or proportion. Develop a margin of error through the use of simulation models for random sampling.

Language Objective
Students will be able to explain how the mean of a sampling distribution, the corresponding population distribution, and the population proportion are related.

Vocabulary
New: sampling distribution, standard error of the mean, standard error of the proportion

Lesson Materials
graphing technology

Module 20: Make Inferences From Data

Recommended Pacing: 6 days

Module 20 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • defined probability distributions and used them to model real-world experiments. • used normal distributions to find probabilities and make decisions. • examined sampling distributions and used them to make predictions about populations. • calculated confidence intervals and margins of error for population proportions and population means.	Students: • identify likely population proportions. • find a confidence interval for a population proportion and population mean. • choose a sample size. • recognize different forms of statistical research. • detect errors in surveys. • identify treatment and control groups. • evaluate a media report of statistical research. • define, formulate, and test a null hypothesis. • perform a resampling and use it to simulate a permutation test.	Students: • will perform paired t-tests to determine statistical significance in comparing two related variables. • will perform tests of correlation, such as a chi-squared test, to determine the strength of an association between two variables.

Module 20 Academic Vocabulary

alternative hypothesis	the difference between the control group and treatment group is due to the treatment
confidence interval	an approximate range of values that is likely to include an unknown population parameter
experiment	an activity in which researchers manipulate one variable by imposing a treatment on some of the subjects of the experiment in order to determine if the treatment has an effect on another variable
margin of error	half the length of the confidence interval
null hypothesis	any measured difference between the control and treatment groups is due to chance
observational study	uses observation to determine whether an existing condition, called a factor, in a population is related to a characteristic of interest. The subjects of the study and their environments are not controlled in any way.
randomized comparative experiment	randomly assigns subjects to one of two groups: the treatment group, which is given the treatment, and the control group, which is not
resampling	to test whether the results of an experiment might be coincidental; combine the original data, randomly sort it into new “treatment” and “control groups,” and find the difference of means
survey	a data collection tool that uses questions to measure characteristics of interest about a population using a sample selected from the population

Lesson 20.1 Confidence Intervals and Margins of Error – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision

I Can Objective

I can calculate a confidence interval and a margin of error for a population proportion or population mean.

Learning Objective

Calculate and interpret a confidence interval of a specific level for a population proportion and mean. Find and interpret the margin of error and determine an appropriate sample size.

Language Objective

Explain how margin of error is related to a confidence interval.

Vocabulary

Review: confidence interval, margin of error

Lesson Materials: grid paper

Lesson 20.2 Surveys, Experiments, and Observational Studies – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- Evaluate reports based on data.

Mathematical Practices and Processes

- Attend to precision.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can identify different types of statistical research and evaluate reports based on statistical research.

Learning Objective

Identify surveys, experiments, and observational studies. Critique statistical research reports.

Language Objective

Explain how to identify types of statistical research studies and how to evaluate those studies.

Vocabulary

New: biased questions, characteristic of interest, control group, experiment, factor, interview effect, nonresponse, observational study, randomized comparative experiment, survey, treatment, treatment group

Lesson Materials: Word description (Teacher Resource Masters), index cards

Lesson 20.3 Making Inferences from Experimental Data – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Mathematical Practices and Processes

- Attend to precision.
- Use appropriate tools strategically.
- Construct viable arguments.

I Can Objective

I can identify when an observed difference between the control group and treatment group in an experiment is likely to be caused by the treatment.

Vocabulary

Review: null hypothesis, alternative hypothesis, statistical significance, resampling, permutation test, p–value

Lesson Materials

notecards, computer

Learning Objective

Use confidence intervals and margins of error to make inferences in real-world surveys, experiments, and observational studies.

Language Objective

Students should be able to describe a null and alternative hypothesis, interpret the mean and standard error, and determine if the data is statistically significant.