HMH Into AGA Geometry

Unit 1: Essentials of Geometry

Unit 1 Project: STEM Task: Wildlife Conservationist – The Birds and the Trees
Unit 1 Learning Mindset Focus: Perseverance: Sustains Attention

Module 1: Geometry in the Plane

Recommended Pacing: 8 days

Module 1 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • wrote and solved linear, multi-step equations in one-variable. • used terms such as ray, angle, and vertex. • showed that the product of a number and its multiplicative inverse (reciprocal) is 1. • applied the Pythagorean Theorem to calculate the length of one side of a triangle given the lengths of the two other sides.	Students: • measure and construct segments. • name and classify angles. • use a protractor to measure and draw angles. • bisect angles and line segments. • write and solve equations about segments and angles. • find the perimeters and area of polygons. • find areas and perimeters of irregular shapes. • apply the Distance Formula to find the distance between points in the coordinate plane.	Students: • will prove theorems involving segments. • will prove theorems involving angles. • will prove theorems about parallel and perpendicular lines. • will prove triangles are congruent or similar. • will identify polygons based on the properties of their segments and angles. • will prove statements about polygons and circles plotted in the coordinate plane. • will transform geometric figures in the coordinate plane.

Module 1 Academic Vocabulary

line	a straight path of points in a plane; has no thickness; continues forever in both directions
supplementary angles	a two angles whose measures have a sum of 180°
plane	a flat surface; has no thickness; extends forever in all directions
point	a geometric figure that names a location; has no dimension
angle	a geometric figure formed by two line segments or rays that share the same endpoint
bisect	to divide into two congruent parts
Distance Formula	distance between two points (x 1 , y 1) and (x 2 , y 2) on the coordinate

	plane is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) and (x_2, y_2) are the endpoints of the segment
midpoint	the point that divides a segment into two congruent segments
postulate	a statement that is accepted as true without proof

Lesson 1.1 Points, Lines, and Planes – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Mathematical Practices and Processes

- Attend to precision.
- Use appropriate tools strategically.
- Reason abstractly and quantitatively.

I Can Objective

I can copy and add segments.

Learning Objective

Understand precise geometric notation, bisect a segment using a compass and straightedge, and apply the Midpoint Formula to solve problems in the coordinate plane involving distance.

Language Objective

Explain the steps for performing a construction with a compass and straightedge.

Vocabulary

Review: line, plane, point

New: bisect, collinear, congruent, coplanar, distance, endpoint, line segment, midpoint, postulate, ray, undefined term

Lesson Materials

compass, straightedge, toothpicks

Lesson 1.2 Define and Measure Angles – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Mathematical Practices and Processes

- Attend to precision.

I Can Objective

I can copy and measure angles.

Learning Objective

Name and classify angles, measure and draw angles using a protractor, construct an angle bisector using a compass, and write and solve equations to solve mathematical problems involving angle relationships.

Language Objective
Explain the steps needed to construct an angle bisector.

Vocabulary
Review: acute angle, complementary angles, obtuse angle, ray, reflex angle, right angle, straight angle, supplementary angles
New: adjacent angles, angle, angle bisector, vertex

Lesson Materials
ruler, compass, protractor

Lesson 1.3 Polygons and Other Figures in the Plane– 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.
- Use appropriate tools strategically.
- Model with mathematics.
- Reason abstractly and quantitatively.

I Can Objective
I can identify and measure a polygon.

Learning Objective
Find the perimeter and area of polygons.

Language Objective
Explain how to use polygons to model real-world shapes, and apply formulas and the Area Addition Postulate to estimate perimeter and area.

Vocabulary
Review: area, nonpolygon, polygon, rectangle, regular polygon, trapezoid, triangle. New: n-gon

Lesson Materials
ruler, drawing compass

Lesson 1.4 Apply the Distance Formula– 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the Distance Formula.
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Mathematical Practices and Processes

- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can measure the distance between two points on the coordinate plane.

Learning Objective

Find the perimeter and the area of a figure on the coordinate plane using the Distance Formula, and model irregular figures with simple polygons to estimate perimeter and area.

Language Objective

Explain how the Distance Formula is used to find the perimeter and the area of a figure on the coordinate plane.

Vocabulary

New: Distance Formula

Lesson Materials

geometric drawing tool, graph paper

Module 2: Tools for Reasoning and Proof

Recommended Pacing: 8 days

Module 2 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • solved basic linear equations and literal equations. • used linear equations and inequalities in one variable to model and solve real-world problems. • solved inequalities in one variable and graphed the solutions. • graphed linear equations and linear inequalities in two variables. • solved systems of linear equations. • solved quadratic equations in one variable.	Students: • define and write conditional and biconditional statements. • use conditional statements to establish whether statements are true or false. • define and contrast inductive and deductive reasoning. • use deductive reasoning to write and understand proofs. • apply the properties of congruence to relationships among segments and angles. • prove theorems about segments and angles.	Students: • will apply and prove theorems about angles to prove lines in the plane are parallel or perpendicular. • will apply theorems about segments and angles to prove triangles are congruent or similar. • will apply theorems about segments and angles to establish polygonal properties and theorems. • apply relationships between segments and angles in right triangles to establish the trigonometric ratios.

Module 2 Academic Vocabulary

linear pair	a pair of adjacent angles whose noncommon sides are opposite rays
vertical angles	nonadjacent angles formed by two intersecting lines
conditional statement	a statement that can be written in the form “if p, then q,” where p is the hypothesis of the statement, and q is the conclusion
contrapositive	a statement formed by both exchanging and negating the hypothesis and conclusion of a conditional statement; if not q, then not p.
converse	a statement formed by exchanging the hypothesis and conclusion of a conditional statement; if q, then p.
deductive reasoning	the use of facts, definitions, and logic to prove a statement is true
Inductive reasoning	showing that a statement is true by looking at a specific case or cases
inverse	a statement formed by negating the hypothesis and conclusion of a conditional statement; if not p, then not q.
postulate	a statement that is accepted as true and does not need to be proven
proof	an argument that uses factual statements and logic to arrive at a conclusion
theorem	a statement that can be proved to be true

Lesson 2.1 Write Conditional Statements – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can write conditional statements and related conditional statements.

Learning Objective

Write a conditional statement and related conditional statements, and determine whether the statements are true.

Language Objective

Write a biconditional statement from a mathematical definition.

Vocabulary

New: biconditional statement, conditional statement, conjecture, contrapositive, converse, counterexample, definition, inverse

Lesson 2.2 Use Inductive and Deductive Reasoning – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can apply deductive reasoning in a mathematical context.

Learning Objective

Differentiate between inductive and deductive reasoning and apply deductive reasoning in the context of geometric proofs.

Language Objective

Explain the logic behind the two-column proof format

Vocabulary
New: deductive reasoning, inductive reasoning, proof, theorem

Lesson Materials
spreadsheet software

Lesson 2.3 Write Proofs about Segments – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively.

I Can Objective
I can use properties of segments to show congruence.

Learning Objective
Use congruence and the Segment Addition Postulate to complete proofs about segments.

Language Objective
Explain the meaning of the properties of congruent segments and the Segment Addition Postulate.

Vocabulary
New: symbolic notation

Lesson 2.4 Write Proofs About Angles – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Prove theorems about lines and angles. Theorems include: vertical angles are congruent.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective
I can use definitions and relations between lines and angles to prove theorems involving lines and angles.

Learning Objective
Apply proof concepts to situations and theorems involving angles.

Language Objective
Explain why the given reasons for the steps in a proof are logical and make sense.

Vocabulary
Review: linear pair, vertical angles

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Unit 2: Parallel and Perpendicular Lines

Unit 2 Project: STEM Task: Textile Engineer – Fiber for Flight
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Sources of Help

Module 3: Lines and Transversals

Recommended Pacing: 5 days

Module 3 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • established facts about the angles created when parallel lines are cut by a transversal. • used postulates to prove theorems about lines and angles (Angle Addition Postulate, Supplementary Angles, Linear Pair Postulate, and Vertical Angles Theorem). • used a straightedge and compass to copy an angle. • used a straightedge and compass to bisect a segment and angle. • explained a proof of the Pythagorean Theorem and its converse.	Students: • construct and identify angle pairs formed by transversals. • identify, explain, and prove the relationship between angle pairs formed when a transversal crosses parallel lines. • prove whether two lines are parallel. • define and construct the perpendicular bisector of a line segment as the set of points that are equidistant from its endpoints.	Students: • will use properties of lines and angles to justify geometric constructions. • will use properties of angles formed when a transversal crosses parallel lines to prove theorems about triangles. • will use properties of parallel lines to represent translations. • will use properties of perpendicular lines to represent reflections. • will use properties of perpendicular lines to prove theorems about triangles. • will construct a circle that circumscribes a triangle.

Module 3 Academic Vocabulary

transversal	a line that intersects two or more coplanar lines at different points
parallel lines	lines in the same plane that do not intersect
perpendicular bisector	a line perpendicular to a segment at the midpoint of the segment
perpendicular lines	lines that intersect at 90° angles

Lesson 3.1 Parallel Lines Crossed by a Transversal – 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Know precise definitions of angle, circle, perpendicular line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- Prove theorems about lines and angles.

Mathematical Practices and Processes

- Attend to precision.
- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can determine the relationship between angle pairs formed by a transversal crossing parallel lines.

Learning Objective

Identify, explain, and prove the relationships formed when a transversal crosses parallel lines.

Language Objective

Explain how angle pairs formed by a transversal intersecting two lines change when the lines are parallel or not parallel.

Vocabulary

Review: alternate interior angles, alternate exterior angles, consecutive interior angles, consecutive exterior angles, corresponding angles, transversal, parallel lines
New: flow proof

Lesson Materials

Tracing paper

Lesson 3.2 Prove Lines are Parallel – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about lines and angles.
- Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can ensure that two lines are parallel by construction.

Learning Objective

Students will be able to prove whether or not two lines are parallel.

Language Objective
Use language related to parallel lines cut by transversal postulates, postulate converses, and angle pairs.

Lesson Materials
compass, straightedge, translucent paper, index cards

Lesson 3.3 Prove Lines Are Perpendicular – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Prove theorems about lines and angles.
- Make formal geometric constructions with a variety of tools (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)
- Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.

I Can Objective
I can ensure that a line is a perpendicular bisector of a segment by construction.

Learning Objective
Students will be able to define and construct the perpendicular bisector of a line segment as the set of points that are equidistant from its endpoints.

Language Objective
Explain how you can ensure that a line is a perpendicular bisector of a segment.

Vocabulary
New: perpendicular, perpendicular bisector

Lesson Materials
compass and straightedge, geometry software

Module 4: Lines on the Coordinate Plane

Recommended Pacing: 6 days

Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • explained why the slope is the same between any two distinct points on a nonvertical line. • understood that lines with the same slope are parallel. • wrote equations of lines in slope-intercept form. • proved the Pythagorean Theorem. • wrote an equation of a line given two points.	Students: • use slope to identify, write, and use equations of parallel and perpendicular lines. • prove the Distance Formula and use it to prove congruence of segments on the coordinate plane. • determine the coordinates of the midpoint of a segment in the coordinate plane. • apply the Distance Formula to find the length of a segment.	Students: • will investigate properties of circles graphed in the coordinate plane. • will investigate transformations in the coordinate plane. • will write coordinate proofs about triangle relationships. • will write coordinate proofs about parallelograms.

Module 4 Academic Vocabulary

coordinate proof	a style of proof where generalized coordinates are used to prove geometric theorems
perpendicular lines	two lines in the same plane that intersect to form 90°angles
slope of a nonvertical line	the ratio *m* of the vertical change (the *rise*) to the horizontal change (the *run*) between any two points on a line

Lesson 4.1 Slope and Equations of Parallel Lines – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

I Can Objective

I can find the equation of a line that is parallel to a given line.

Learning Objective

Use slope to identify, write, and use equations of parallel lines.

Language Objective

Explain the steps need to write an equation of a line parallel to a given line that passes through a given point.

Vocabulary

Review: coordinate proof, slope

Lesson Materials

ruler

Lesson 4.2 Slope and Equations of Perpendicular Lines – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can use slope to write the equation of a line that is perpendicular to a given line.

Learning Objective

Write the equation of a line that is perpendicular to a given line.

Language Objective

Explain the generalization that can be made about the slopes of perpendicular lines.

Vocabulary

Review: perpendicular lines

Lesson 4.3 Write a Coordinate Proof – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Use coordinates to prove simple geometric theorems algebraically.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively.

I Can Objective

I can use the Distance Formula to show congruence on the coordinate plane

Learning Objective

Students will use coordinates to prove simple geometric theorems algebraically.

Language Objective

Given two points, students should be able to explain the process of how to find the distance between the points using the Distance Formula and the Pythagorean Theorem and how to find the midpoint of the line connecting the points using the Midpoint Formula.

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Unit 3: Transformations

Unit 3 Project: STEM Task: Pulmonologist – Making Some Breathing Room
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 5: Transformations That Preserve Size and Shape

Recommended Pacing: 8 days

Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • verified the properties of reflections experimentally. • learned that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. • described the effect of reflections on two-dimensional figures using coordinates	Students: • measure and construct segments in the coordinate plane. • define a translation, rotation, and reflection as a function that preserves measures of segments and angles. • draw the image of a translated, rotated, or reflected figure. • describe rotations and reflections that carry a given figure onto itself. • examine the properties of symmetry in the plane.	Students: • will define dilations, stretches, and skews. • will apply sequences of transformations. • will identify congruent triangles and polygons. • will prove triangles are congruent if and only if pairs of corresponding sides and angles are congruent. • will apply the ASA, SAS, and SSS theorems to prove triangle congruence. • will use HL and AAS to prove right triangle congruence.

Module 5 Academic Vocabulary

angle of rotational symmetry	the smallest angle of rotation that maps a figure to itself
image	the corresponding points of a figure after a transformation of a preimage
isometry	a rigid motion
line of symmetry	a line that divides a plane figure into two congruent reflected halves
rigid motion	a transformation that does not change the size or shape of a figure
rotation	a rigid motion that turns a figure about a point P, such that each point and its image are the same distance from P
rotational symmetry	a rotation of a figure about its center by an angle of 180° or less so that the image coincides with the preimage
transformation	a function that changes the position, size, or shape of a figure or graph
vector	a quantity that has both direction and magnitude

Lesson 5.1 Define and Apply Translations – 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Represent transformations in the plane using, e.g., transparencies and geometry; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation versus horizontal stretch).
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices and Processes

- Attend to precision.
- Use appropriate tools strategically.

I Can Objective

I can translate figures in the plane.

Learning Objective

Develop a definition of translation as a function that preserves measures of segments and angles and draw the image of a figure under such a transformation.

Language Objective

Explain how to determine the location of the image of a translation relative to the preimage when given the translation vector in component form.

Vocabulary

New: component form, image, isometry, preimage, rigid motion, transformation, translation, vector

Lesson Materials

Lesson Materials: compass, straightedge

Lesson 5.2 Define and Apply Rotations – 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Represent transformations in the plane using, e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can rotate figures in the plane.

Learning Objective
Develop a definition of rotation as a function that preserves measures of segments and angles and draw the image of a figure under such a transformation.

Language Objective
Explain how the properties of circles are used when finding the image of a figure rotated about a center by an angle.

Vocabulary
New: center of rotation, rotation

Lesson Materials
compasses, geometry software, protractors, straightedges, tracing paper/transparencies

Lesson 5.3 Define and Apply Reflections – 1 Day

Focus on:
Connect Concepts and Skills

Mathematics Standards
- Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices and Processes
- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.

I Can Objective
I can reflect figures in a plane.

Learning Objective
Develop a definition of reflection as a function that preserves measures of segments and angles and draw the image of a figure under such a transformation.

Language Objective
Describe properties of reflection and the steps for reflecting figures on the coordinate plane using mathematical language

Vocabulary
Review: line of reflection, reflection

Lesson Materials
tracing paper, compass, straightedge, mirrors, patterns blocks

Lesson 5.4 Define and Apply Symmetry – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity with repeated reasoning.

I Can Objective

I can identify symmetry in figures

Learning Objective

Describe the rotations and reflections that carry a given figure onto itself.

Language Objective

Explain how to determine the number of lines of symmetry and the angle of rotational symmetry for any regular polygon.

Vocabulary

New: angle of rotational symmetry, line of symmetry, line symmetry, rotational symmetry, symmetry

Module 6: Transformations That Change Size and Shape

Recommended Pacing: 4 days

Module 6 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • learned the definitions of rotations, reflections, and translations in the plane. • described the effects of translations, rotations, and reflections on figures in the x coordinate plane. • learned that two-dimensional figures are similar if the second is the image of the first through a rotation, reflection, or translation. • represented and described transformations in the plane as functions that take points as inputs and yield points as outputs. • compared transformations that preserve distances and angles to those that do not.	Students: • extend transformations to include dilations and stretches. • compare rigid and nonrigid transformations. • apply sequences of transformations to move figures in the plane. • specify a transformational sequence that maps a preimage to an image. • predict the result of applying a sequence of transformations.	Students: • will use similarity transformations to determine whether two figures are similar. • will apply similarity transformations to establish the AA Theorem and prove that two triangles are similar. • will use similarity to define the trigonometric ratios.

Module 6 Academic Vocabulary

dilation	a transformation that changes the size of a figure by the same amount in all directions
center of dilation	a fixed point in the plane that does not change when a dilation is applied
composition	a transformation that directly maps a preimage to the final image after each image is used as a preimage in the next transformation.
compression	a transformation that changes the shape of a figure in one direction by a factor greater than 0 and less than 1
scale factor	the ratio of the length of a segment on the image to the length of the corresponding segment on the preimage
stretch	a transformation that changes the shape of a figure by a factor greater than 1 in one direction

Lesson 6.1 Define and Apply Dilations, Stretches, and Compressions – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can dilate and stretch a figure and determine how a figure has been transformed.

Learning Objective

Perform and analyze transformations to include dilations, stretches, and compressions. Use coordinate rules and geometric drawing tools to investigate the effect of multiplication on the points in a figure

Language Objective

Compare and contrast a dilation with a stretch or a compression

Vocabulary

Review: dilation

New: center of dilation, compression, scale factor, stretch

Lesson Materials

compass, geometric drawing tool, straightedge

Lesson 6.2 Slope and Equations of Perpendicular Lines – 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices and Processes

- Attend to precision.
- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can determine the effects of a sequence of transformations on a figure.

Learning Objective

Apply sequences of transformations to figures, specify sequences that map a given preimage to a given image, and make predictions about the result of applying a sequence of transformations.

Language Objective

Understand and use the language of transformations and compositions of transformations.

Vocabulary

New: composition

Lesson Materials

index cards

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Unit 4: Triangle Congruence

Unit 4 Project: STEM Task: Architect – Coordinating Congruencies
Unit 4 Learning Mindset Focus: Resilience: Responds to Feedback

Module 7: Congruent Triangles and Polygons

Recommended Pacing: 6 days

Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • understood that 2-dimensional figures are congruent if the 2nd can be obtained from the 1st by a sequence of rigid motions. • represented transformations in the plane. • developed definitions of rotations, reflections, and translations. • specified transformations that map given figure onto another. • used rigid motions to transform figures.	Students: • use rigid motions to show figures are congruent. • use congruency of corresponding parts to prove triangles are congruent. • write proofs involving congruent figures.	Students: • will write coordinate proofs about triangle relationships. • will write coordinate proofs about parallelograms. • will identify and describe relationships among lines and angles of a circle, including the relationships among central, inscribed, and circumscribed angles. • will construct inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Module 7 Academic Vocabulary

congruent	two figures such that one can be obtained from the other by a sequence of rigid motions
corresponding angles	angles in the same position in polygons with an equal number of sides
corresponding sides	sides in the same position in polygons with an equal number of sides
vector	a quantity that has both direction and magnitude

Lesson 7.1 Understand Congruent Figures– 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices and Processes

- Attend to precision.
- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can determine whether figures are congruent.

Learning Objective

Use rigid motions to show figures are congruent and find unknown measures in congruent figures.

Language Objective

Explain how to determine if two figures are congruent using rigid motions or corresponding angles and sides

Vocabulary

Review: congruent

New: corresponding angles, corresponding sides

Lesson Materials

tracing paper, ruler, protractor

Lesson 7.2 Corresponding Parts of Congruent Figures – 1 Day

Focus on:

Apply and Practice

Mathematics Standards

- Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

I Can Objective

I can use congruent figures to solve problems.

Learning Objective

Use congruent figures to identify congruent parts of figures, solve for unknown measures, and prove geometric statements.

Language Objective

Explain how knowing two figures are congruent can help us solve for unknown measures of parts of the figures.

Vocabulary
Review: corresponding angles, corresponding sides, biconditional, converse, contrapositive

Lesson Materials
rulers, protractor, scissors, geometry tiles, GeoGebra

Lesson 7.3 Use Rigid Motions to Prove Figures Are Congruent – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

Learning Objective
Use the definition of congruence in terms of rigid motions to determine if two given figures are congruent.

I Can Objective
I can use rigid motions to show that figures are congruent.

Language Objective
Explain how you know if an image is congruent to its preimage and describe transformations that resulted in the image.

Module 8: Triangle Congruence Criteria

Recommended Pacing: 8 days

Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • represented transformations in the plane and described transformations as functions. • developed definitions of rotations, reflections, and translations. • used rigid motions to transform figures. • used definitions of rigid motions to determine congruency. • proved triangle congruence by rigid motions.	Students: • use rigid motions to show figures are congruent. • use congruency of corresponding parts to prove triangles are congruent. • use ASA, SSS, SAS, AAS, and HL congruence criteria to prove that two triangles are congruent.	Students: • will write coordinate proofs about triangle relationships. • will write coordinate proofs about parallelograms. • will use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. • will construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Module 8 Academic Vocabulary

included angle	an angle formed by two sides of a triangle
included side	a side connecting the vertices of two angles

Lesson 8.1 Develop ASA Triangle Congruence – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics

I Can Objective

I can use ASA congruence criteria to prove that two triangles are congruent.

Learning Objective

Students use ASA congruence criteria to prove that two triangles are congruent.

Language Objective

Explain the steps needed to prove triangles congruent using ASA congruence criteria.

Vocabulary

New: included side

Lesson Materials

ruler, protractor, geometry software

Lesson 8.2 Develop SAS Triangle Congruence – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can use SAS congruence criteria to prove that two triangles are congruent.

Learning Objective

Define and identify SAS triangle congruence criteria and prove they are sufficient to assume triangle congruence and use SAS congruence criteria to prove triangles are congruent.

Language Objective

Use corresponding sides and angles and SAS congruence to define congruent triangles.

Vocabulary

New: included angle

Lesson Materials

compass, protractor, scissors, ruler

Lesson 8.3 Develop SSS Triangle Congruence – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

I Can Objective

I can use SSS congruence criteria to prove that two triangles are congruent.

Learning Objective

Analyze triangle congruency using the SSS criteria through constructions, rigid transformations, formal proofs, and inspection of diagrams. Solve problems related to triangles that are congruent by SSS.

Language Objective

Describe when two triangles are congruent by identifying three pairs of corresponding pairs of congruent sides.

Lesson Materials

compass, ruler, strips of paper

Lesson 8.4 Develop SSS AAS and HL Triangle Congruence – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can use AAS and HL congruence criteria to determine if triangles are congruent.

Learning Objective

Use HL and AAS congruence criteria to determine if triangles are congruent.

Language Objective

Explain the difference between HL congruence criteria and the congruence criteria required for other triangle congruency theorems.

Lesson Materials

protractor, compass, ruler, index cards

HMH Into AGA Geometry

Unit 5: Relationships Within Triangles

Unit 5 Project: STEM Task: Environmental Chemist – Room for Bloom
Unit 5 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 9: Properties of Triangles

Recommended Pacing: 10 days

Module 9 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • proved triangle congruence criteria. • proved lines are perpendicular and constructed perpendicular bisectors of segments. • proved theorems about perpendicular bisectors.	Students: • prove theorems about perpendicular bisectors and angle bisectors of triangles. • construct angle bisectors. • use constructions to find the incenter and circumcenter of a triangle. • prove that the altitudes of a triangle meet at a point. • prove that the medians of a triangle meet at a point. • prove and use the Triangle Midsegment Theorem.	Students: • will prove theorems about triangle inequalities. • will prove theorems about quadrilaterals.

Module 9 Academic Vocabulary

isosceles triangle	a triangle with at least two congruent sides
altitude of a triangle	the perpendicular segment from a vertex to the opposite side or to a line that contains the opposite side
centroid of a triangle	the point of concurrency of the medians
circumcenter of a triangle	the point of concurrency of the perpendicular bisectors
incenter of a triangle	the point of concurrency of the angle bisectors of a triangle
median of a triangle	the segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side
midsegment of a triangle	the segment that joins the midpoints of two sides of a triangle
orthocenter of a triangle	the point of concurrency of the altitudes of the triangle
perpendicular bisector of a side of a triangle	the segment that is perpendicular to and bisects a side of a triangle

Lesson 9.1 Angle Relationships in Triangles – 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.

I Can Objective

I can prove theorems about triangle angles.

Learning Objective

Students prove theorems about triangle angles and apply the theorems in solving problems.

Language Objective

Students construct arguments and prove theorems about angle relationships using precise language.

Vocabulary

Review: complementary angles, isosceles triangle, supplementary angles New: auxiliary line, corollary, exterior angle, interior angle,
remote interior angle

Lesson Materials

protractor, Bubble Map (Teacher Resource Masters), Index Cards, straws, tape

Lesson 9.3 Angle Bisectors in Triangles – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.

I Can Objective

I can prove that angle bisectors are concurrent and inscribe circles in triangles.

Learning Objective

Students prove the Angle Bisector Theorem, the converse of the Angle Bisector Theorem, and the Incenter Theorem, and students construct the incenters and inscribed circles of triangles.

Language Objective
Describe the intersection of the angle bisectors of a triangle as a point called the incenter that is equidistant from the sides of the triangle and the center of the inscribed circle of a triangle.

Vocabulary
New: angle bisector of a triangle, incenter

Lesson Materials: compass, straightedge, ruler

Lesson 9.3 Angle Bisectors in Triangles – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Mathematical Practices and Processes

- Attend to precision.
- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.

I Can Objective
I can prove that angle bisectors are concurrent and inscribe circles in triangles.

Learning Objective
Students prove the Angle Bisector Theorem, the converse of the Angle Bisector Theorem, and the Incenter Theorem, and students construct the incenters and inscribed circles of triangles.

Language Objective
Describe the intersection of the angle bisectors of a triangle as a point called the incenter that is equidistant from the sides of the triangle and the center of the inscribed circle of a triangle

Vocabulary
New: angle bisector of a triangle, incenter

Lesson Materials
compass, straightedge, ruler

Lesson 9.4 Medians and Altitudes in Triangles – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.

I Can Objective

I can construct medians and altitudes to find centroids and orthocenters.

Learning Objective

Students will construct medians and altitudes to find centroids and orthocenters.

Language Objective

Discuss the basics of centroids and orthocenters and explain how to find them.

Lesson Materials

Straight edge

Lesson 9.5 The Triangle Midsegment Theorem – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can construct midsegments and prove the Triangle Midsegment Theorem.

Learning Objective

Construct midsegments of a given triangle, prove the Triangle Midsegment Theorem, and apply the theorem to solve for segment lengths and angle measures.

Language Objective

Explain how we can use the Triangle Midsegment Theorem to find the coordinates of the endpoints of a midsegment of a triangle from the coordinates of the vertices of the triangle.

Vocabulary

New: midsegment of a triangle

Lesson Materials

compass, ruler, protractor, straightedge

Module 10: Triangle Inequalities

Recommended Pacing: 2 days

Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • investigated the relationship between an exterior angle and the remote interior angles of a triangle. • used the Triangle Sum Theorem. • wrote and solved inequalities.	Students: • learn how to apply inequalities to relate the sides and angles in one triangle. • relate the sides and angles in two triangles.	Students: • will investigate segment length and angle measure relationships among lines intersecting circles.

Module 10 Academic Vocabulary

indirect proof	a proof whose assumption is that the conclusion is false and shows that this assumption leads to a contradiction

Lesson 10.1 Inequalities Within a Triangle – 1 Day

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Reason abstractly and quantitatively.

I Can Objective

I can determine the relative sizes of angles and sides in a triangle.

Learning Objective

Decide when three lengths can form a triangle, find the possible ranges of side lengths for the third side of a triangle, and order and compare the side lengths and angle measures in a triangle.

Language Objective

Explain how to determine when given side lengths describe a triangle.

Lesson Materials

straightedge, compass

Lesson 10.2 Inequalities Between Two Triangles – 1 Day

Focus on:

Apply and Practice

Mathematics Standards

- Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangles is parallel to the third side and half the length; the medians of a triangle meet at a point.
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can determine the relative sizes of angles and sides in two triangles.

Learning Objective

Apply known information about a triangle(s) in a pair of triangles to determine relative lengths and angle measures.

Language Objective
Explain the difference between the Hinge Theorem and the Converse of the Hinge Theorem.

Vocabulary
Review: indirect proof

Lesson Materials: Paper strips

HMH Into AGA Geometry

Unit 6: Quadrilaterals, Polygons, and Triangle Similarity

Unit 6 Project: STEM Task: Digital Animator – Meshing Around
Unit 6 Learning Mindset Focus: Resilience: Adjusts to Change

Module 11: Quadrilaterals and Polygons

Recommended Pacing: 10 days

Module 11 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • proved and used properties of triangles. • proved that lines are parallel. • proved properties of parallelograms.	Students: • prove and use properties of and conditions for parallelograms, rectangles, rhombuses, and squares. • prove and apply theorems about trapezoids and kites.	Students: • will construct inscribed quadrilaterals and identify properties of their angles

Module 11 Academic Vocabulary

base angles of a trapezoid	the pairs of consecutive angles whose common side is a base of the trapezoid
bases of a trapezoid	the sides of a trapezoid that are parallel
isosceles trapezoid	a trapezoid in which the legs are congruent but not parallel
kite	a quadrilateral whose four sides can be grouped into two pairs of consecutive congruent sides
legs of a trapezoid	the sides that are not the bases
midsegment of a trapezoid	the segment whose endpoints are the midpoints of the legs of a trapezoid
parallelogram	a quadrilateral with both pairs of opposite sides parallel
rectangle	a parallelogram with four right angles
rhombus	a parallelogram with four congruent sides
trapezoid	a quadrilateral with at least one pair of parallel sides

Lesson 11.1 Properties of Parallelograms – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can prove and use properties of parallelograms.

Learning Objective

Students prove and use properties of parallelograms.

Language Objective

Explain the properties of parallelograms.

Vocabulary

New: diagonal of a polygon, parallelogram

Lesson Materials

dynamic software, compass, straightedge, straws, protractor

Lesson 11.2 Conditions for Parallelograms– 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

I Can Objective

I can prove and use conditions for parallelograms.

Learning Objective

Students prove and use conditions for parallelograms.

Language Objective

Explain the steps needed to prove conditions necessary to prove a quadrilateral is a parallelogram.

Lesson 11.3 Properties of Rectangles, Rhombuses, and Squares – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure

I Can Objective

I can prove and use properties of squares, rectangles, and rhombuses.

Learning Objective

Students will prove theorems about rectangles, rhombuses, and squares, discriminate among them based on their properties, and apply theorems to find measures of sides, diagonals, and interior angles.

Language Objective

Students write and explain the definitions of rectangles, rhombuses, and squares and describe their properties.

Vocabulary

New: rectangle, rhombus, square

Lesson Materials

geometry software

Lesson 11.4 Conditions for Rectangles, Rhombuses, and Squares – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals.*
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- Use coordinates to prove simple geometric theorems algebraically.
- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.)

Mathematical Practices and Processes

- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can prove and use conditions for rectangles, rhombuses, and squares.

Learning Objective
Students use the properties of diagonals for rectangles, rhombuses, and squares to identify special parallelograms in the coordinate plane and construct two-step proofs of theorems to prove conditions for rectangles and rhombuses.

Language Objective
Students describe the properties of diagonals for rectangles, rhombuses, squares, and the conditions for rectangles and rhombuses.

Lesson 11.5 Properties and Conditions for Trapezoids and Kites – 2 Days

Focus on:
Apply and Practice

Mathematics Standards
- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Mathematical Practices and Processes
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Reason abstractly and quantitatively.

I Can Objective
I can prove and apply theorems about trapezoids and kites.

Learning Objective
Use the exclusive and inclusive definitions of kite and trapezoid to classify quadrilaterals, prove and apply theorems about angle measures and diagonals of kites, prove and apply theorems about conditions on a trapezoid equivalent to it being isosceles, and apply the Trapezoid Midsegment Theorem.

Language Objective
Explain how different types of quadrilaterals can be considered kites or trapezoids.

Vocabulary
New: base angles of a trapezoid, bases of a trapezoid, isosceles trapezoid, kite, legs of a trapezoid, midsegment of a trapezoid, trapezoid

Module 12: Similarity

Recommended Pacing: 8 days

Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • performed dilations and sequences of transformations.	Students: • use corresponding parts of similar triangles to solve problems. • prove and use the Triangle Proportionality Theorem. • identify similar right triangles. develop the AA similarity criterion for triangles.	Students: • will explore trigonometry with right triangles. • will explore trigonometry with all triangles.

Module 12 Academic Vocabulary

geometric mean	the second and third terms in the proportion _ ax = _ x b where the first and fourth terms are a and b; equal to the positive value of √ a— b
partition	divide a segment into smaller segments
Pythagorean triple	a set of three nonzero whole numbers that satisfy the Pythagorean Theorem
similar figures	figures that have the same shape but not necessarily the same size
similarity transformation	a transformation in which an image has the same shape as the preimage

Lesson 12.1 Use Transformations to Prove Figures are Similar – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- Prove that all circles are similar.

Mathematical Practices and Processes

- Attend to precision.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can use similarity transformations to prove figures are similar.

Learning Objective

Determine when figures are similar using transformations and comparing corresponding side ratios. Solve problems to prove figures are similar and to find missing values.

Language Objective

Explain how to determine if two figures are similar.

Vocabulary

New: similar figures, similarity transformation

Lesson Materials

geometric drawing tool

Lesson 12.2 Develop AA Triangle Similarity– 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can prove AA, SSS, and SAS Similarity Theorems

Learning Objective
Prove the AA Triangle Similarity Theorem and use it to find missing dimensions of triangles. Use the SSS and SAS Triangle Similarity Theorems to prove triangles are similar and find missing dimensions.

Language Objective
Explain how to prove the AA Triangle Similarity Theorem.

Lesson Materials
Cards for Sharpen Skills and Plan for Differentiated Instruction activities

Lesson 12.3 Develop and Prove Triangle Proportionality – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
- Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Use appropriate tools strategically.

I Can Objective
I can identify and use the connection between parallel lines and proportional segments in triangles

Learning Objective
Analyze a proof of the Triangle Proportionality Theorem, apply the theorem to solve for lengths of partitions of triangle sides, apply the converse of the theorem to determine partitions of triangle sides that give a line parallel to another side, and find the point on a directed line segment that partitions the segment in a given ratio.

Language Objective
Explain how the Triangle Proportionality Theorem applies to triangles and how it can be used to solve for unknown lengths.

Vocabulary
New: partition

Lesson Materials
compass, ruler

Lesson 12.4 Apply Similarity in Right Triangles – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
- Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices and Processes

- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.

I Can Objective

I can identify similar right triangles, apply the Geometric Means Theorems, and recognize Pythagorean triples.

Learning Objective

Identify similar right triangles created when an altitude is drawn to the hypotenuse of a right triangle, apply the Geometric Means Theorems, use geometric means to prove the Pythagorean Theorem, and recognize Pythagorean triples.

Language Objective

Explain how you can find the lengths of the segments of the hypotenuse of a right triangle when you draw an altitude to the hypotenuse.

Vocabulary

New: geometric mean, Pythagorean triple

HMH Into AGA Geometry

Unit 7: Right Triangle Trigonometry

Unit 7 Project: STEM Task: Surveyor – Sound Surveying
Unit 7 Learning Mindset Focus: Perseverance: Learns Effectively

Module 13: Trigonometry with Right Triangles

Recommended Pacing: 8 days

Module 13 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used the definition of similarity in terms of similarity transformations to decide if triangles are similar. • used congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Students: • use trigonometric ratios and their inverses to find side lengths and angle measures in right triangles. • use trigonometric ratios and the Pythagorean Theorem to find the side lengths and angle measures of special right triangles. • solve right triangles in applied problems.	Students: • will derive the formula A = _ 1_ 2 ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Module 13 Academic Vocabulary

cosine	in right triangle ABC with the right angle at C, cos A = length of leg adjacent ∠A / length of hypotenuse = AC/AB
inverse cosine	in right triangle ABC with the right angle at C, sin -1 (a/c) = m∠A
inverse sine	in right triangle ABC with the right angle at C, cos -1 (b/c) = m∠A
sine	in right triangle ABC with the right angle at C, sin A = length of leg opposite ∠ A/ length of hypotenuse= BC/AB
tangent	the ratio of the length of the side opposite an angle in a right triangle to the length of the side adjacent to the angle
trigonometric ratio	a ratio of two sides of a right triangle

Lesson 13.1 Tangent Ratio – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Attend to precision.

I Can Objective

I can use the tangent ratio and the inverse tangent to find side lengths and angle measures in right triangles.

Learning Objective

Solve for missing side lengths and angle measures using the tangent, inverse tangent, and properties of similar triangles.

Language Objective

Explain which sides of a right triangle are used for the tangent ratio.

Vocabulary

New: inverse tangent, tangent

Lesson Materials

calculator, geometric drawing tool, tracing paper

Lesson 13.2 Sine and Cosine Ratios – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- Explain and use the relationship between the sine and cosine of complementary angles.
- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Mathematical Practices and Processes

- Attend to precision.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

I Can Objective

I can use sine and cosine ratios and the inverses to find side lengths and angle measures in right triangles.

Learning Objective

Use sine and cosine ratios and the inverses to find side lengths and angle measures in right triangles.

Language Objective

Use trigonometric terminology appropriately and with understanding.

Vocabulary
New: cosine, inverse cosine, inverse sine, sine, trigonometric ratio

Lesson Materials
calculator, geometric drawing tool

Lesson 13.3 Special Right Triangles – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- Explain and use the relationship between the sine and cosine of complementary angles.
- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure
- Use appropriate tools strategically.

I Can Objective
I can use trigonometric ratios and the Pythagorean Theorem to find the side lengths and angle measures of special right triangles.

Learning Objective
Examine and use trigonometric ratios for special right triangles, and use the Pythagorean Theorem to find the side lengths and angle measures of special right triangles.

Language Objective
Explain how to find the ratio of the sides of special right triangles.

Vocabulary
New: special right triangles

Lesson Materials
compass, geometric drawing tool, protractor, ruler

Lesson 13.4 Solve Problems Using Trigonometry – 2 Days

Focus on:
Apply and Practice

Mathematics Standards

- Explain and use the relationship between the sine and cosine of complementary angles.
- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective
I can use trigonometric ratios, the area formula for a triangle in terms of its side lengths, and the Pythagorean Theorem to solve right triangles in applied problems.

Learning Objective

Apply the sine, cosine, and tangent ratios and the inverses to find the areas of triangles and measures of sides and acute angles in right triangles both in the coordinate plane and represented in real-world situations.

Language Objective

Explain how to derive the area formula A = _ 1_ 2 ab sin C and how to solve problems involving right triangles and trigonometric ratios.

Vocabulary

New: angle of depression, angle of elevation, solve a right triangle

Lesson Materials

calculator

Module 14: Trigonometry With All Triangles

Recommended Pacing: 4 days

Module 14 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • applied the Pythagorean Theorem to determine unknown side lengths in right triangles. • understood that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios.	Students: • use the Law of Sines to find side lengths and angle measures of non-right triangles and solve real-world problems. • use the Law of Cosines to find side lengths and angle measures of non-right triangles and solve real-world problems.	Students: • will identify and describe relationships among inscribed angles, radii, and chords. • will construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.

Lesson 14.1 Law of Sines – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove the Laws of Sines and Cosines and use them to solve problems.
- Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Mathematical Practices and Processes

- Look for and make use of structure.
- Use appropriate tools strategically.
- Model with mathematics.

I Can Objective

I can use the Law of Sines to find side lengths and angle measures of non-right triangles and solve real-world problems.

Learning Objective

Prove the Law of Sines, determine which combinations of given triangle measures are sufficient to apply the Law of Sines, determine how many triangles can exist with an ambiguous (SSA) combination of given measures, and apply the Law of Sines to determine unknown triangle measures.

Language Objective

Explain how more than one triangle can have a given SSA combination of measures.

Lesson Materials

calculator

Lesson 14.2 Law of Cosines – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Prove the Laws of Sines and Cosines and use them to solve problems.
- Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
- Derive the formula A = _ 1_ 2 ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
- Attend to precision.

I Can Objective

I can use the Law of Cosines to find side lengths and angle measures of non-right triangles and solve real-world problems.

Learning Objective

Derive the Law of Cosines, recognize the SAS and SSS cases where the Law of Cosines is applicable and apply it to find unknown measures, derive the area formula A = 1/2 ab sin C for a triangle, and find the area of triangles when given an SAS or SSS combination of measures.

Language Objective

Explain when you can use the Law of Cosines to find an unknown measure in a triangle, and when you can use the Law of Sines.

Lesson Materials

calculator

HMH Into AGA Geometry

Unit 8: Properties of Circles

Unit 8 Project: STEM Task: Optical Lens Technician – Looking Through the Right Lens
Unit 8 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 15: Angles and Segments in Circles

Recommended Pacing: 8 days

Module 15 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • applied the Pythagorean Theorem to determine unknown side lengths in right triangles. • completed the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. • developed definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. • wrote coordinate proofs about triangle relationships.	Students: • define and determine the measures of central angles, inscribed angles, and arcs of a circle. • use the properties of angles of quadrilaterals inscribed in a circle to prove theorems and solve problems. • prove theorems about tangents to a circle and use them to solve mathematical and real-world problems. • derive and write the equation of a circle with radius r and center (h, k).	Students: • will identify and describe relationships among inscribed angles, radii, and chords, including the relationship between central, inscribed, and circumscribed angles. • will derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality. • will derive the formula for the area of a sector

Module 15 Academic Vocabulary

circle	the set of all points in a plane that are equidistant from a given point called the center
diameter	a chord of a circle that contains the center of the circle
arc	an unbroken part of a circle consisting of two points called endpoints and all points on the circle between them
central angle	an angle whose vertex is the center of the circle
chord	a segment whose endpoints lie on a circle
circumscribed angle	an angle formed by two rays from a common endpoint that are tangent to a circle
inscribed angle	an angle whose vertex is on a circle and whose sides are chords of the circle

intercepted arc	an arc that consists of its endpoints and all points of the circle between the endpoints
tangent of a circle	a line in the same plane as the circle that intersects the circle in exactly one point

Lesson 15.1 Central Angles and Inscribed Angles – 2 Days

Focus on:

Build Conceptual Understanding

Mathematics Standards

- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can determine the measures of central angles, inscribed angles, and arcs of a circle.

Learning Objective

Identify the relationships between the measures of associated central angles, inscribed angles, and intercepted arcs; apply the Arc Addition Postulate; apply the Inscribed Angle Theorem to solve for unknown arc and angle measures.

Language Objective

Describe the relationships of angles and arcs of circles.

Vocabulary

Review: circle, diameter

New: adjacent arcs, arc, central angle, chord, inscribed angle, intercepted arc, major arc, minor arc, semicircle

Lesson Materials

compass, geometric drawing tool, straightedge

Lesson 15.2 Angles in Inscribed Quadrilaterals – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.

I Can Objective
I can use the properties of angles of quadrilaterals inscribed in a circle to prove theorems and solve problems.

Learning Objective
Students will be able to prove the Inscribed Quadrilateral Theorem and the Congruent Corresponding Chords Theorem and apply them to quadrilaterals inscribed in circles.

Language Objective
Students will be able to explain steps in the proofs of the theorems for quadrilaterals covered in the lesson and the way they're applied to specific examples.

Vocabulary
New: congruent circles, congruent arcs

Lesson Materials
protractor, ruler, compass

Lesson 15.3 Tangents and Circumscribed Angles – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards

- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.
- Attend to precision.

I Can Objective
I can prove theorems about tangents to a circle and use them to solve mathematical and real-world problems.

Learning Objective
Solve problems where tangent lines form angles with other tangent lines and with the radius of the circle. Prove that two tangent segments that share an exterior point are congruent. Prove that a circumscribed angle is supplementary to its related central angle. Prove that a tangent is perpendicular to the radius it intersects.

Lesson Materials
compass, geometric drawing tool, straightedge

Language Objective
Explain how to prove and use the properties of tangents to a circle.

Vocabulary
New: circumscribed angle, exterior of a circle, interior of a circle, point of tangency, tangent of a circle

Lesson 15.4 Circles on the Coordinate Plane – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- Use coordinates to prove simple geometric theorems algebraically.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can derive and write the equation of a circle with radius r and center (h, k).

Learning Objective

Students will write an equation of a circle given its radius and the coordinates of its center, complete the square to rewrite an equation of a circle so its center and radius can be easily identified, and determine whether a given point lies on a circle given the equation of the circle.

Language Objective

Students will be able to explain the process of writing the equation of the circle given its radius and the coordinates of its center.

Lesson Materials

compass, geometric drawing tool, straightedge, graph paper

Module 16: Relationships in Circles

Recommended Pacing: 4 days

Module 16 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • proved theorems about lines, angles, and triangles. • used congruence and similarity criteria of triangles to solve problems and to prove relationships. • identified relationships among inscribed angles, radii, and chords of circles.	Students: • prove theorems about the relationships of chords, secants, and tangents of circles. • use segment and angle relationships in circles to solve mathematical and real world problems. • prove theorems about angle relationships of circles.	Students: • justify the formulas for the circumference and area of a circle.

Module 16 Academic Vocabulary

external secant segment	a secant segment that lies in the exterior of a circle with one point on the circle
secant	a line that intersects a circle at exactly two points
secant segment	a segment of a secant line with at least one endpoint on the circle
tangent segment	a segment of a tangent with one endpoint on the circle

Lesson 16.1 Segment Relationships in Circles – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify and describe relationships among inscribed angles, radii, and chords*. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

I Can Objective

I can use segment relationships in circles to solve mathematical and real-world problems.

Vocabulary

New: external secant segment, secant, secant segment, tangent segment

Learning Objective

Use proportional relationships in circles to prove the Chord-Chord, Secant-Secant, and Secant-Tangent Product Theorems. Apply the theorems to solve for segment lengths in mathematical and real-world problems.

Language Objective

Explain how to derive the equation of the Chord-Chord Product Theorem from the relationship of similar triangles

Lesson Materials geometric drawing tool

Lesson 16.2 Angle Relationships in Circles – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can use angle relationships in circles to solve mathematical and real-world problems.

Learning Objective

Determine the relationships that exist between secants, tangents, and chords in a circle and the angles and arcs formed by them. Prove and use theorems about these relationships to solve mathematical and real-world problems.

Language Objective

Describe the differences in the relationships of arcs in circles when the intersecting secants, tangents, and chords are inside, on or outside the circle.

Module 17: Circumference and Area of a Circle

Recommended Pacing: 6 days

Module 17 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used the formulas for the circumference and area of a circle to solve problems. • used trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. • proved that all circles are similar. • identified relationships among inscribed angles, radii, and chords of circles.	Students: • justify the formulas for the circumference and area of a circle. • use the formulas for the circumference and area of a circle to solve mathematical and real-world problems. • derive and use the formula for arc length and area of a sector to solve mathematical and real-world problems. • convert between degree and radian measure.	Students: • will use geometric shapes, their measures, and their properties to describe three-dimensional objects. • will find the volumes and surface areas of cylinders, pyramids, cones, and spheres. • will use the formulas for volume and surface area to solve problems. • will explain how the unit circle in the coordinate plane can be used to extend the trigonometric functions to all real numbers.

Module 17 Academic Vocabulary

circumference	the distance around a circle
arc length	the distance along an arc, measured in linear units, such as centimeters
concentric circles	coplanar circles that have the same center
limit	a value that the output of a function approaches as the input increases or decreases without bound or approaches a given value
radian measure	the ratio of the length of a circular arc to the radius of the arc
sector	a portion of a circle bounded by two radii and their intercepted arc

Lesson 17.1 Measure Circumference and Area of a Circle – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics

I Can Objective

I can justify and use the circumference and area of a circle formulas to solve real-world problems.

Learning Objective

Justify and use the formulas for the circumference and area of a circle to solve real-world and mathematical problems

Language Objective

Explain two different ways to find the circumference of a circle when given the measure of its radius.

Vocabulary

Review: circumference

New: limit

Lesson Materials

Spreadsheet

Lesson 17.2 Measure Arc Length and Use Radians – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can use similarity of circles to find arc length.

Learning Objective

Use the arc length formula and apply it to real-world problems, and convert between degree and radian measure.

Language Objective

Explain how to derive the arc length formula using similarity of circles.

Vocabulary

New: arc length, concentric circles, radian measure.

Lesson 17.3 Measure Sector Area – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can use sector area to solve real-world problems

Learning Objective

Derive the formula for the area of a sector of a circle and use that formula to compute the area of sectors of circles having different central angles and radii.

Language Objective

Explain using correct terminology how to find the area of a sector of a circle.

Vocabulary

New: sector

HMH Into AGA Geometry

Unit 9: Surface Area and Volume

Unit 9 Project: STEM Task: Naval Architect – Tremendous Tanks
Unit 9 Learning Mindset Focus: Strategic Help-Seeking

Module 18: Surface Area

Recommended Pacing: 8 days

Module 18 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • represented three-dimensional figures using nets. • used nets to find the surface area of three-dimensional figures. • solved mathematical and real-world problems involving area, volume, and surface area.	Students: • identify solids of rotation. • identify cross sections of solids. • use nets to develop formulas for surface area. • use formulas for the surface areas of solids to solve problems. • find surface areas of hemispheres and composite figures.	Students: • will translate between the geometric description and the equation for a conic section. • for a function that models a relationship between two quantities, will interpret key features of graphs and tables in terms of the quantities, and sketch the graphs showing key features given a verbal description of the relationship.

Module 18 Academic Vocabulary

cross section	the intersection of a three dimensional figure and a plane
hemisphere	half of a sphere
lateral area	the sum of the areas of the lateral faces of a three-dimensional figure
net	a diagram of a three-dimensional figure arranged in such a way that the diagram can be folded to form the three-dimensional figure
oblique solid	a solid in which the axis or the lateral edges are not perpendicular to the base(s)
population density	the number of organisms of a particular type per square unit of area
right solid	a solid in which the axis or the lateral edges are perpendicular to the base(s)
slant height	the height of each lateral face of a regular pyramid
solid of rotation	a solid that is formed by rotating a shape about an axis
surface area	the total area of all faces and curved surfaces of a three-dimensional figure

Lesson 18.1 Three-Dimensional Figures – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can identify the characteristics of three-dimensional figures and represent them using drawings.

Learning Objective

Identify and classify three-dimensional solids by name, identify solids of rotation with plane figures rotated about axes, and identify cross sections of solids in planes parallel and not parallel to bases.

Language Objective

Explain when a cross section of a solid is a transformation of a base of the solid.

Vocabulary

New: cone, cross section, cylinder, oblique solid, prism, pyramid, right solid, solid of rotation, sphere

Lesson 18.2 Surface Areas of Prisms and Cylinders – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics
- Attend to precision.

I Can Objective

I can find the surface area of a prism or cylinder.

Learning Objective

Develop the formulas for the surface areas of right prisms and right cylinders and use the formulas to solve mathematical problems. Apply surface area to population density problems in the real world.

Language Objective

Describe how the net of a figure relates to the formula for surface area.

Vocabulary
Review: composite figure

New: lateral area, net, population density, surface area

Lesson 18.3 Surface Areas of Pyramids and Cones – 2 Days

Focus on:
Connect Concepts and Skills

Mathematics Standards
- Use geometric shapes, their measures, and their properties to describe objects.
- Apply concepts of density based on area and volume in modeling situations.
- Apply geometric methods to solve design problems.

Mathematical Practices and Processes
- Look for and make use of structure
- Model with mathematics.
- Attend to precision.

I Can Objective
I can use formulas for the surface area of pyramids and cones to solve real-world problems.

Learning Objective
Use formulas for the surface area of pyramids and cones to solve real-world problems.

Language Objective
Explain why a pyramid that is not regular does not have a slant height.

Vocabulary
New: regular pyramid, slant height, right cone

Lesson 18.4 Surface Areas of Spheres – 2 Days

Focus on:
Apply and Practice

Mathematics Standards
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematical Practices and Processes
- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective
I can use the formula for the surface area of a sphere to calculate the surface areas of composite figures.

Learning Objective

Find the surface area of a sphere and use the formula to find the surface area of hemispheres and composite figures in real-world problems.

Language Objective

Explain how the lateral area of a cylinder (containing a sphere of the same radius that intersects each base of the cylinder in exactly one point) and the surface area of the sphere are related.

Vocabulary

New: hemisphere

Module 19: Volume

Recommended Pacing: 6 days

Module 19 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used formulas for the volumes of cones, cylinders, prisms, pyramids, and spheres to solve real-world and mathematical problems. • solved real-world and mathematical problems involving the areas of two-dimensional objects composed of known shapes. • solved real-world and mathematical problems involving the surface area of three-dimensional objects.	Students: • develop, relate, and apply formulas for the volumes of right and oblique prisms and cylinders. • algebraically model the volumes of rectangular prisms and use a graphing calculator to find the maximum volume and the associated dimensions. • apply a formula for volume to solve a real-world problem involving density. • find the volume of composite figures involving spheres. • estimate volume in a real-world situation.	Students: • will interpret key features for a function that models a relationship between two quantities. • will sketch graphs showing key features of a function given a verbal description of the relationship.

Module 19 Academic Vocabulary

volume	the number of non-overlapping cubic units contained within a three-dimensional figure

Lesson 19.1 Volumes of Prisms and Cylinders – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Identify and describe relationships among inscribed angles, radii, and chords*. Include the*
- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's Principle, and informal limit arguments.
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.
- Use appropriate tools strategically.
- Model with mathematics.

I Can Objective

I can develop and use formulas for the volume of a prism and a cylinder

Learning Objective

Develop, relate, and use formulas for the volumes of prisms and cylinders. Use algebraic models and a graphing calculator to maximize the volumes of rectangular prisms.

Language Objective

Explain how to find the volume of an oblique prism or cylinder.

Vocabulary

Review: volume

Lesson Materials

Graphing calculator

Lesson 19.2 Volumes of Pyramids and Cones – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
- Apply concepts of density based on area and volume in modeling situations (e.g., person per square mile, BTUs per cubic foot).
- Use geometric shapes, their measures, and their properties to describe objects (e.g. modeling a tree trunk or a human torso as a cylinder).

Mathematical Practices and Processes

- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Attend to precision.

I Can Objective
I can show the relationship between the volume formulas for pyramids and cones problems.

Learning Objective
Derive formulas for the volume of a cone and a pyramid, relating the formulas to the volumes of cylinders and prisms; and solve real-world and mathematical problems by finding the volumes of pyramids, cones, and composite figures.

Language Objective
Describe how the volume formulas for a cone and pyramid are derived from the volume formulas of prisms and cylinders.

Lesson 19.3 Volumes of Spheres – 2 Days

Focus on:

Apply and Practice

Mathematics Standards

- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective
I can use the formula for the volume of a sphere to calculate the volumes of composite figures.

Learning Objective
Derive and use a formula for the volume of a sphere. Use the volume formula to solve real-world problems including calculating capacity and dimensions. Calculate the volume of composite figures involving hemispheres and other known solids.

Language Objective
Explain how the volume of a sphere and a hemisphere are related to each other and to a cylinder.

HMH Into AGA Geometry

Unit 10: Probability

Unit 10 Project: STEM Task: Genetic Counselor – It's Probably Genetic
Unit 10 Learning Mindset Focus: Perseverance: Collects and Tries Multiple Strategies

Module 20: Probability of Multiple Events

Recommended Pacing: 4 days

Module 20 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • expressed the likelihood of an event as a number between 0 and 1. • used positive and negative numbers to represent quantities in real-world contexts. • summarized categorical data in two-way frequency tables. • interpreted relative frequencies in the context of the data.	Students: • understand how elements and subsets relate to a larger universal set. • describe and represent sets, their characteristics, and their relationships. • calculate theoretical probabilities. • devise the Addition Rule for the probability of the union of overlapping events. • calculate probabilities of unions of events	Students: • will describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events. • will understand that two events are independent if the probability of the events occurring together is the product of their probabilities. • will understand the conditional probability of an event occurring given that another event has already occurred.

Module 20 Academic Vocabulary

complement	complement for a given event, all the outcomes that are not part of the event; The sum of the probabilities of an event and its complement must equal 1.
disjoint	when two events that cannot occur in the same trial at the same time; another word for mutually exclusive
element	element each object in a set
empty set	a set with no elements, denoted by ∅ or { }
intersection of two sets	the set of elements that belong to both sets
mutually exclusive	two events that cannot occur in the same trial at the same time; another term for disjoint
set	a collection of distinct objects

subset of set A	a set in which all elements of the set are also contained within set A
union of two sets	the set of elements that belong to one or both sets
universal set	the set of all elements in a particular context

Lesson 20.1 Probability and Set Theory – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively.
- Attend to precision

I Can Objective

I can use sets and their relationships to understand and calculate probabilities.

Learning Objective

Describe sets and their relationships, including the universal set and complement, calculate theoretical probabilities and outcomes of events, and use the complement of an event to calculate probability.

Language Objective

Use precise language to describe sets and calculate probabilities.

Vocabulary

Review: event, outcome, probability experiment, sample space, theoretical probability, trial. New: complement, element, empty set, intersection, set, subset, union, universal set

Lesson 20.2 Disjoint and Overlapping Events – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- Apply the Addition Rule, P (A or B) = P(A) + P(B) - P(A and B) , and interpret the answer in terms of the model.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

I Can Objective

I can calculate probabilities for both disjoint and overlapping events.

Learning Objective

Recognize disjoint and overlapping events and understand the Addition Rule to find the probability of the union of overlapping events.

Language Objective

Explain in everyday language what it means for events to be mutually exclusive.

Vocabulary

New: disjoint events, mutually exclusive events, overlapping events

Module 21: Conditional Probability and Independence of Events

Recommended Pacing: 6 days

Module 21 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • summarized and interpreted categorical data in two-way frequency tables • interpreted relative frequencies in the context of the data. • used two-way frequency tables to determine if events are independent and to determine approximate conditional probabilities.	Students: • find conditional probabilities from a two-way table. • derive and apply the Conditional Probability Formula. • use the Multiplication Rule to find the probability of events. • determine whether two events are • independent.	Students: • will describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events. • will understand that two events are independent if the probability of the events occurring together is the product of their probabilities. • will understand the conditional probability of an event occurring given that another event has already occurred.

Module 21 Academic Vocabulary

conditional probability	given two events A and B, the probability that event B occurs given that event A has already occurred
dependent event	an event affected by occurrence of another event
independent event	an event that is not affected by occurrence of another event
relative frequency	the frequency of one outcome divided by the frequency of all outcomes

Lesson 21.1 Conditional Probability – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- Understand the conditional probability of A given B as (A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Mathematical Practices and Processes

- Look for and make use of structure
- Attend to precision.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can calculate conditional probability and use it to solve real-world problems.

Learning Objective

Calculate conditional probability and use it to solve real-world problems.

Language Objective

Explain in speaking or in writing the meaning of conditional probability and how it can be calculated.

Vocabulary

New: conditional probability, relative frequency

Lesson 21.2 Independent Events – 2 Days

Focus on:

Connect Concepts and Skills

Mathematics Standards

- Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision

I Can Objective

I can determine whether two events are independent and find their probabilities

Learning Objective
Interpret independence and its definition in terms of conditional probability. Derive and apply the Multiplication Rule for the probability of the intersection of independent events and use the rule to test for independence of events. Use the concept of independence to solve real-world problems.

Language Objective
Explain what it means for two events to be independent in terms of a conditional probability

Vocabulary
New: independent events

Lesson 21.3 Dependent Events – 2 Days

Focus on:
Apply and Practice

Mathematics Standards
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A) P(B \mid A) = P(B) P(A \mid B)$, and interpret the answer in terms of the model.

Mathematical Practices and Processes
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and express regularity in repeated reasoning.

I Can Objective
I can determine whether two events are dependent and find their probabilities.

Learning Objective
Determine whether two events are dependent and find their probabilities.

Language Objective
Explain how to determine whether two events are dependent and find their probabilities.

Vocabulary
New: dependent events