## HMH Into AGA Algebra 1

### **Unit 1: Real Numbers and Connections to Algebra**

**Unit 1 Project**: STEM Task: Chemist – Solution Dilution **Unit 1 Learning Mindset Focus**: Resilience: Monitors Knowledge and Skills

### **Module 1: Real Numbers and Real-World Quantities**

Recommended Pacing: 5 days

#### **Module 1 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>identified rational and irrational.</li> <li>performed operations with rational numbers.</li> <li>Applied the properties of integer exponents.</li> <li>Evaluated square roots of small perfect squares and cube roots of small perfect cubs.</li> <li>Used place value to round decimals.</li> </ul>	<ul> <li>Students:</li> <li>classify real numbers.</li> <li>understand closure properties of rational numbers under addition and multiplication.</li> <li>simplify expressions involving radicals and rational exponents.</li> <li>learn the difference between precision and accuracy.</li> <li>select an appropriate level of precision when reporting quantities.</li> </ul>	<ul> <li>Students:</li> <li>will model and solve problems involving radicals, radical equations, and radical functions.</li> <li>will understand the continuous nature of exponential functions.</li> <li>will make calculations with measurement data and communicate their findings precisely,</li> <li>will report any limitations on their findings based on their data collection method.</li> </ul>

#### Module 1 Academic Vocabulary

index	a value written over the root symbol
traditional number	a number that cannot be expressed as the ratio of two integers
radical expression	an expression that contains the radical symbol $\ $
radicand	an expression or value under the root symbol
rational number	a number that can be expressed as the ratio of two integers
real numbers	the set of all rational and irrational numbers
accuracy	the closeness of a measured value to the actual
closure	a set of numbers has closure under a given operation if the result of the operation on any two numbers in the set is also in the set
precision	the level of detail of a measurement, determined by the smallest unit or fraction of a unit that can be reasonably measured
significant digits	the digits in a measurement that carry a meaning in terms of the precision of the measurement
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### Lesson 1.1 Real Numbers – 1 Day

#### Focus on:

Build Conceptual Understanding

#### Mathematics Standards

• Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

#### **Mathematical Practices and Processes**

- Look for and make use of structure
- Construct viable arguments and critique the reasoning of others.

#### I Can Objective

I can operate with real numbers and understand closure properties in the real number system

#### **Learning Objective**

Review the structure of the real number system, including a Venn diagram of sets

within the real number system, and closure properties of rational numbers under addition and multiplication.

#### Language Objective

Explain how a number fits into different sets within the real number system.

#### Vocabulary

Review: irrational number, rational number, real number New: closure

### Lesson 1.2 Radicals and Rational Exponents – 2 Days

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### **Mathematical Practices and Processes**

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

#### I Can Objective

I can simplify expressions involving radicals and rational exponents.

#### **Learning Objective**

Understand rational exponents and simplify expressions involving radicals and rational exponents.

#### Language Objective

Explain the steps used to simplify an expression with rational exponents.

#### Vocabulary

Review: index, radical expression, radicand



### **Lesson 1.3 Precision and Accuracy in Calculations** – 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

• Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

#### **Mathematical Practices and Processes**

- Attend to precision
- Use appropriate tools strategically

#### I Can Objective

I can report quantities with accuracy and an appropriate level of precision.

#### **Learning Objective**

Explain the difference between precision and accuracy, and select an appropriate level of precision when reporting quantities.

#### Language Objective

Explain the precision of a measurement

#### Vocabulary

New: accuracy, precision, significant digit



# Module 2: Linear Equations and Inequalities in One Variable Recommended Pacing: 7 days

#### **Module 2 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>applied the properties of arithmetic to simplify numerical expressions.</li> <li>understood that the sum of a number and its additive inverse is 0.</li> <li>understood that the product of a number and its multiplicative inverse is 1.</li> <li>solved basic linear equations in one variable.</li> <li>verified solutions of equations by substituting values for the variable.</li> </ul>	<ul> <li>Students:</li> <li>classify real numbers.</li> <li>create and simplify algebraic expressions.</li> <li>solve linear equations in one variable.</li> <li>solve literal equations to create new formulas and use the general solutions to find specific solutions.</li> <li>solve simple and compound inequalities in one variable and graph the solutions on a number line.</li> <li>use linear equations and inequalities in one variable to model and solve real-world problems.</li> </ul>	<ul> <li>Students:</li> <li>will graph linear equations in two variables when the equations are given in different forms.</li> <li>will solve systems of linear equations using substitution and linear combinations.</li> <li>will graph linear inequalities in two variables.</li> <li>will solve a literal equation to produce the quadratic formula.</li> <li>will solve quadratic equations in one variable.</li> </ul>

### Module 2 Academic Vocabulary

equation	a mathematical statement indicating that two expressions are equal by using the symbol =
equivalent equations	equations that have the same solution
expression	a mathematical phrase that combines numbers and/or variables using mathematical operations
inequality	a mathematical statement indicating that two expressions are unequal by using one of the symbols <, >, $\leq$ , or $\geq$
solution of equation (or inequality) in one variable	a number that, when substituted for the variable in the equation (or inequality), produces a true statement
compound inequality	two or more inequalities joined by and or <i>or</i>
literal equation	an equation in which constants have been replaced by letters



## **Lesson 2.1 Write, Interpret, and Simplify Expressions** – 1 Day Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

#### I Can Objective

I can write an algebraic expression, interpret the parts of the expression, and use the Distributive Property to simplify the expression.

#### **Learning Objective**

Write, interpret, and simplify linear expressions in one variable, and use linear expressions and compatible units to model real-world situations.

#### Language Objective

Explain how to write and interpret a linear expression that models a real-world situation and use the Distributive Property to simplify the expression.

#### Vocabulary

Review: coefficient, equivalent expressions, expression, like terms, term

## Lesson 2.2 Write and Solve Equations – 1 Day Focus on:

Apply and Practice

#### **Mathematics Standards**

- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

#### **Mathematical Practices and Processes**

- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics.
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#### I Can Objective

I can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides.

#### **Learning Objective**

Solve linear equations with grouping symbols or with the variable on both sides, and use linear equations to model and solve real-world problems.

#### Language Objective

Explain the steps needed to solve a linear equation with grouping symbols or with the variable on both sides.



#### Vocabularv

Review: equation, equivalent equations, solution of an equation in one variable

## Lesson 2.3 Precision and Accuracy in Calculations – 2 Days

Focus on:

Apply and Practice

#### **Mathematics Standards**

- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.

#### I Can Objective

I can solve both simple and more complicated literal equations for a given variable.

#### **Learning Objective**

**Lesson Materials** 

Spreadsheet software

Rewrite formulas to express a variable of interest in terms of the other variables, and solve literal equations to obtain a general solution for a class of equations.

#### Language Objective

Explain the steps needed to solve a literal equation for a given variable.

#### Vocabulary

New: literal equation

#### Lesson 2.4 Write and Solve Inequalities- 1 Day Focus on:

Apply and Practice

#### **Mathematics Standards**

- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### **Mathematical Practices and Processes**

- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics.

#### I can write inequalities to model real-world problems and use properties of inequality to solve the inequalities.

#### **Learning Objective**

Write and solve linear inequalities in one variable, represent solutions of linear



#### I Can Objective

inequalities on a number line, and use linear inequalities to model and solve real-world problems.

#### Language Objective

Explain how to write and solve a linear inequality that models a real-world situation

#### Vocabulary

Review: inequality, solution of an inequality in one variable

**Lesson Materials** Spreadsheet software

## **Lesson 2.5 Write and Solve Compound Inequalities** – 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

- Create equations and inequalities in one variable and use them to solve problems.
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can write, solve, and graph compound inequalities, and use compound inequalities to model real-world problems.

#### **Learning Objective**

Write and solve compound linear inequalities in one variable using both "and" and "or," represent solutions of compound inequalities on a number line, and use compound inequalities to model and solve real-world problems.

#### Language Objective

Explain how to write and solve compound inequalities.

#### Vocabulary

New: compound inequality



## HMH Into AGA Algebra 1

### **Unit 2: Linear Functions and Equations**

**Unit 2 Project**: STEM Task: Aerospace Engineer – Up In Thin Air **Unit 2 Learning Mindset Focus**: Perseverance: Sustains Focus

### Module 3: Linear Equations in Two Variable

Recommended Pacing: 3 days

#### **Module 3 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>wrote and simplified expressions</li> <li>wrote and solved equations</li> <li>graphed linear equations</li> <li>interpreted the equation y = mx + b as a linear function whose graph is a line, where m is the slope and b is the y-intercept.</li> </ul>	<ul> <li>Students:</li> <li>connect linear equations in standard form to their graphs</li> <li>identify intercepts for linear equations</li> <li>model with linear equations</li> <li>calculate and interpret slope</li> <li>use slope to graph lines</li> </ul>	<ul> <li>Students:</li> <li>will write and interpret linear functions.</li> <li>will study linear models and other forms of linear functions.</li> <li>will calculate and interpret the average rate of change of a function.</li> <li>will graph linear functions.</li> </ul>

#### Module 3 Academic Vocabulary

linear equation in	an equation that can be written in the standard form of a linear equation	
rate of change	a ratio of the amount of change in the dependent variable to the amount of change in the independent variable	
slope	a measure of the steepness of a line	
standard form of a linear equation	Ax + By = C, where A, B, and C are constants and A and B are not both zero	
x-intercept	the x-coordinate of the point where the graph of a relation intersects the x-axis	
y-intercept	the y-coordinate of the point where the graph of a relation intersects the y-axis	



## **Lesson 3.1 Linear Equations in Standard Form –** 2 Days

#### Focus on:

Build Conceptual Understanding

#### Mathematics Standards

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

#### **Mathematical Practices and Processes**

- Look for and express regularity in repeated reasoning.
- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.
- Reason abstractly and quantitatively.

#### I Can Objective

I can connect solutions of equations to points on their graphs.

#### **Learning Objective**

Graph linear equations given in standard form and connect solutions of the equations to points on their graphs.

#### Language Objective

Describe the process for graphing a linear equation given in standard form.

#### Vocabulary

New: linear equation in two variables, standard form of a linear equation, xintercept, y-intercept

## **Lesson 3.2 Slopes of Lines and Rates of Change**– 1 Day Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from the graph.
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Look for and make use of structure.

#### I Can Objective

I can numerically describe and interpret the slope of a line.

#### **Learning Objective**

Calculate, use, and interpret the slope of a line given any two points on the line.

#### Language Objective

Explain the steps needed to calculate the slope of a line and interpret the slope.

#### Vocabulary

New: rate of change, slope



## Module 4: Linear Functions and Models

Recommended Pacing: 8 days

#### Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>understood that a function is a rule that assigns to each input exactly one output.</li> <li>used linear functions to model real-world situations.</li> <li>determined the slope of a function from information in a table, graph, or equation.</li> <li>interpreted slope in real- world situations.</li> </ul>	<ul> <li>Students:</li> <li>determine if a relation is a function.</li> <li>write functions using function notation.</li> <li>write linear functions in slope-intercept form and in point-slope form.</li> <li>determine whether linear functions are increasing or decreasing.</li> <li>determine end behavior, zeros, and extreme values of linear functions.</li> </ul>	<ul> <li>Students:</li> <li>will compare linear functions.</li> <li>will find inverses of linear functions.</li> <li>will fit linear functions to data.</li> <li>will model with polynomials.</li> </ul>

### Module 4 Academic Vocabulary

continuous function	has a graph connected with no gaps or breaks	
dependent variable	depends on the value of another variable	
discrete function	has a graph made up of unconnected points	
domain	the set of all possible inputs for a relation	
end behavior	describes the trend in y-values as x-values approach positive or negative infinity	
function	a relation in which every domain value is paired with exactly one range value	
independent variable	determines the value of another variable	
linear function	an equation relating the variables x and y, where the graph of the equation is a line	
range	the set of all possible outputs for a relation	
relation	a set of ordered pairs $(x, y)$ , where x represents the input value and y represents the output value	
zero	any value of x such that $f(x) = 0$	



### Lesson 4.1 Relations and Functions- 2 Days

#### Focus on:

Build Conceptual Understanding

#### Mathematics Standards

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

#### **Mathematical Practices and Processes**

• Reason abstractly and quantitatively.

Attend to precision.

#### Learning Objective

Contrast relations and functions, and use function notation.

I Can Objective I can identify how functions and relations Language Objective

Explain how to determine whether a given relation is a function.

#### Vocabulary

New: dependent variable, domain, function, function rule, independent variable, range, relation, vertical line test

### Lesson 4.2 Linear Functions – 2 Days

#### Focus on:

are related.

Build Conceptual Understanding

#### **Mathematics Standards**

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

#### Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.
- Reason abstractly and quantitatively.



#### I Can Objective

I can describe the graphs of linear functions.

#### **Learning Objective**

Define, graph, and analyze linear functions, and identify the slope and y-intercept from equations and tables of values.

#### Language Objective

Explain how to identify and graph linear functions

#### Vocabulary

New: continuous function, discrete function, linear function, slope-intercept form

## **Lesson 4.3 Characteristics of Linear Functions** – 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Create For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Look for and make use of structure.
- Reason abstractly and quantitatively.

#### I Can Objective

I can identify the characteristics of linear functions.

#### **Learning Objective**

Identify the characteristics of linear functions, including end behavior, zeros,

extreme values, and whether they are increasing or decreasing.

#### Language Objective

Explain how to identify the end behavior, zeros, extreme values, and other characteristics of linear functions.

#### Vocabulary

New: decreasing, end behavior, increasing, maximum, minimum, zero

## Lesson 4.4 Linear Models and Point-Slope Form – 2 Days

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features



Houghton Mifflin Harcourt. The Learning Company\* given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

• Determine an explicit expression, a recursive process, or steps for calculation from a context.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can use linear functions to model realworld scenarios.

#### Learning Objective

Write and analyze linear functions to model real-world scenarios.

#### Language Objective

Explain the steps needed to model realworld scenarios using linear functions.

#### Vocabulary

New: point-slope form



# Module 5: Relationships Among Linear Functions Recommended Pacing: 6 days

### Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>Worked with functions and function notation to manipulate functions algebraically.</li> <li>drew connections among graphs, equations, and verbal descriptions.</li> <li>created linear models.</li> <li>graphed and evaluated linear functions.</li> <li>identified functions and their inputs and outputs.</li> <li>evaluated functions for inputs in their domains.</li> <li>used function notation.</li> </ul>	<ul> <li>Students:</li> <li>recognize transformations as changes to the domain and range values.</li> <li>draw connections among transformations in graphs, equations, and verbal descriptions.</li> <li>apply transformations to functions and graphs from their verbal descriptions.</li> <li>create a new function from an existing one by adding a constant function to it.</li> <li>compare linear functions in different forms.</li> <li>find and graph inverses of linear functions.</li> </ul>	<ul> <li>Students:</li> <li>will graph piecewise-defined functions</li> <li>will compare exponential functions.</li> <li>Will perform operations with functions</li> </ul>

#### Module 5 Academic Vocabulary

family of functions	a group of functions that have basic characteristic in common
inverse of a function	maps the output values of a function back to their original input values
inverse function	exists if the inverse of a function f is itself a function, and is denoted by f -1 (read as "f inverse")
parent function	the simplest function with the defining characteristics of the family of functions
transformation	a change in the position, size, or shape of a figure or graph



## **Lesson 5.1 Transform Graphs of Functions** – 2 Days Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Reason abstractly and quantitatively.

#### I Can Objective

I can determine how transformations to functions affect their graphs.

#### **Learning Objective**

Identify basic transformations of functions and their corresponding graphs.

Differentiate between the effects of adding a constant or multiplying by a constant on the domains and ranges of functions.

#### Language Objective

Describe each of the four basic types of transformations, and summarize strategies that can be used to identify each of these types of transformations.

## Vocabulary

New: transformation

## **Lesson 5.2 Transform Linear Functions** – 2 Days Focus on:

Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Identify the effect on the graph of replacing f(x) by f(x) + k, k × f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Write a function that describes a relationship between two quantities.
- Combine standard function types using arithmetic operations.
- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Attend to precision.

• Model with mathematics.



#### I Can Objective

I can relate all linear functions to the linear parent function f(x) = x.

#### **Learning Objective**

Identify the linear parent function and relate all other linear functions to the linear parent function.

#### Language Objective

Describe what the linear parent function is and how transformations affect the graph of the linear parent function.

#### Vocabulary

New: family of functions, parent function

### Lesson 5.3 Compare Linear Functions – 1 Day

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

• Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Attend to precision.

#### I Can Objective

I can compare functions given in different forms

#### **Learning Objective**

Compare linear functions given in different forms, considering their effective uses and emphases.

#### Language Objective

Explain uses and advantages of linear functions given in different forms.

## Lesson 5. 4 Inverses of Linear Functions – 1 Day

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

- Solve an equation of the form f(x) = c for a simple function c that has an inverse and write an expression for the inverse.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Attend to precision.
- Reason abstractly and quantitatively.

#### I Can Objective

I can write an equation for the inverse of a linear function and use inverses of linear functions to solve problems.

#### **Learning Objective**

Find, graph, and confirm inverse functions.

#### Language Objective

Explain how to convert between a function and its inverse.

#### Vocabulary

New: inverse functions, inverse of a function



## HMH Into AGA Algebra 1

### **Unit 3: Build Linear Functions and Models**

**Unit 3 Project**: STEM Task: Geologist – On Shaky Ground **Unit 3 Learning Mindset Focus**: Challenge-seeking: Makes Decisions

### **Module 6: Fit Linear Functions to Data**

Recommended Pacing: 4 days

#### **Module 6 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>created scatter plots with bivariate data and identified the type of association in the data set.</li> <li>informally fit a line to data and assessed the fit of the line.</li> <li>used a linear model of bivariate data to solve problems by interpreting the slope and y-intercept of the linear equation.</li> </ul>	<ul> <li>Students:</li> <li>estimate correlation coefficients.</li> <li>use two points to write a line of fit.</li> <li>consider whether there is a causation for the correlation of a data set.</li> <li>solve problems using a line of fit by interpolation and extrapolation.</li> <li>assess the fit of a function using residuals</li> </ul>	<ul> <li>Students:</li> <li>will construct and compare linear, quadratic, and exponential models and use the models to solve problems.</li> <li>will interpret the parameters in a linear or exponential function in terms of a context</li> </ul>

#### Module 6 Academic Vocabulary

line of best fit	a line that comes closest to all of the points in a data set
line of fit	a line on a scatter plot that helps show the correlation between variables more clearly
scatter plot	a graph with data points plotted
bivariate data	data for two paired variables
correlation	a measure of the strength and direction of a linear relationship between two variables
Correlation coefficient	a statistic that describes how closely the points in a scatter plot cluster about a line
extrapolation	making a prediction using a value of the independent variable outside the model's domain
interpolation	making a prediction using a value of the independent variable from within the model's domain
least-squares line	the line of best fit for which the sum of the squares of the residuals is as small as possible
linear regression	a statistical method used to find the least-squares line for a given data set
residual	the signed vertical distance between a data point and a line of fit
residual plot	a scatter plot of the data points generated by the values of the independent variable and the corresponding residuals



## **Lesson 6.1 Scatter Plots, Correlation, and Fitted Lines** – 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Fit a linear function for a scatter plot that suggests a linear association.
- Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- Distinguish between correlation and causation.

#### **Mathematical Practices and Processes**

- Use appropriate tools strategically.
- Attend to precision.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

#### I Can Objective

I can create a line fitted to data that shows a linear correlation and use it to make predictions.

#### **Learning Objective**

Make a scatter plot of data, determine whether there is a correlation between variables, fit a line to the data, and use a fitted line to make predictions about data that have a strong correlation.

#### Language Objective

Explain the steps required to write an equation of a line of fit.

#### Vocabulary

Review: scatter plot, line of fit New: bivariate data, correlation, correlation coefficient, interpolation, extrapolation

## **Lesson 6.2 Residuals and Best-Fit Lines** – 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- Informally assess the fit of a function by plotting and analyzing residuals.
- Fit a linear function for a scatter plot that suggests a linear association.
- Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- Compute (using technology) and interpret the correlation coefficient of a linear fit.

#### **Mathematical Practices and Processes**

- Use appropriate tools strategically.
- Attend to precision.
- Reason abstractly and quantitatively.

#### I Can Objective

I can use the linear regression function on a graphing calculator to find a line of best fit for a bivariate data set.



#### **Learning Objective**

Enter bivariate data into a graphing calculator, use it to find a fitted linear regression line, determine the strength of and compare lines of fit, and use the equation of a regression line to solve problems.

#### Language Objective

Explain how a correlation coefficient and residual plots can be

used to assess the fit of a data set to a chosen model.

#### Vocabulary

Review: line of best fit. New: least–squares line, linear regression, residual, residual plot

**Lesson Materials:** graphing calculator, spreadsheet software



## **Module 7: Discrete Linear Functions**

Recommended Pacing: 4 days

#### Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>generated number patterns given a rule.</li> <li>identified a rule from a given number pattern.</li> <li>used function notation.</li> <li>modeled real-world situations using linear functions.</li> <li>graphed linear functions.</li> </ul>	<ul> <li>Students:</li> <li>find the nth term in an arithmetic sequence using a recursive rule.</li> <li>model a real-world situation using an arithmetic sequence.</li> <li>write arithmetic sequences using an explicit or recursive formula.</li> <li>identify the domain and range of a sequence and determine whether the sequence is infinite or finite.</li> <li>convert between recursive and explicit forms for arithmetic sequences.</li> </ul>	<ul> <li>Students:</li> <li>will write a recursive formula for a geometric sequence and find the nth term of the geometric sequence given a recursive formula.</li> <li>will write an explicit formula for a geometric sequence and find the nth term of the geometric sequence given an explicit formula.</li> <li>will convert between explicit and recursive forms for geometric sequences.</li> <li>will solve problems using real-world geometric sequences.</li> </ul>

### Module 7 Academic Vocabulary

arithmetic sequence	an ordered list of numbers with successive numbers differing by the same nonzero number
common difference	the difference between one term and the next in an arithmetic sequence
discrete function	a function with a domain that is not continuous and a graph with a disconnected set of points
explicit rule	a rule that defines the nth term as a function of n for any whole number n greater than 0
recursive rule	a rule that defines the nth term of a sequence by relating it to one or more of the previous terms in the series
sequence	a list of numbers in a specific order
term of a sequence	each element or number in a sequence



## **Lesson 7.1 Arithmetic Sequences Defined Recursively**– 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Attend to precision.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can use arithmetic sequences defined recursively to solve real-world problems.

#### Learning Objective

Write a recursive rule that models an arithmetic sequence, and use the recursive rule to solve real-world problems.

#### Language Objective

Explain how to determine the common difference for an arithmetic sequence, and how to use the common difference when writing a recursive rule.

#### Vocabulary

Review: arithmetic sequence, common difference, discrete function, recursive rule, sequence, term (of the sequence)

### Lesson 7.2 Arithmetic Sequences Defined Explicitly – 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

#### **Mathematical Practices and Processes**

- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.

#### I Can Objective

I can use arithmetic sequences defined explicitly to solve real-world problems.



#### **Learning Objective**

Use function notation and an explicit formula to model real world relationships and solve problems involving arithmetic sequences.

#### Language Objective

Explain the steps needed to write a given arithmetic sequence explicitly.

Vocabulary Review: explicit rule



## **Module 8: Piecewise-Defined Functions**

Recommended Pacing: 6 days

#### Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>learned that functions are a type of relation in which each element in the domain is paired with exactly one element in the range.</li> <li>determined whether a given relation is a function.</li> <li>wrote, graphed, and used linear functions to solve real-world problems.</li> <li>solved compound inequalities</li> </ul>	<ul> <li>Students:</li> <li>graph piecewise-defined functions.</li> <li>model real-world situations using piecewise-defined functions.</li> <li>evaluate piecewise-defined functions.</li> <li>graph absolute value functions.</li> <li>use absolute value functions to solve real-world problems.</li> <li>solve absolute value equations and inequalities graphically and algebraically</li> </ul>	<ul> <li>Students:</li> <li>will transform absolute value functions.</li> <li>will solve complex equations involving conjunctions and disjunctions algebraically.</li> </ul>

### Module 8 Academic Vocabulary

greatest integer function	a function that rounds the number x down to the greatest integer that is less than or equal to x	
piecewise-defined function	a function that is a combination of one or more functions	
step function	a piecewise-defined function that is constant over each interval of its domain	
axis of symmetry	a line that divides a plane figure or a graph into two congruent reflected halves	
tolerance	the amount by which a measurement is permitted to vary from a prescribed measure	
vertex of an absolute- value graph	the highest or lowest point on the graph of an absolute value function	



## **Lesson 8.1 Graph Piecewise-Defined Functions** – 2 Days Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

#### **Mathematical Practices and Processes**

- Model with mathematics.
- Look for and make use of structure.
- Attend to precision.

#### I Can Objective

I can write, graph, and use piecewise-defined functions to solve real-world problems.

#### **Learning Objective**

Write, graph, and use piecewise-defined functions to solve real-world problems.

#### Language Objective

Explain methods used to solve and graph piecewise-defined functions, step functions, and greatest integer functions.

#### Vocabulary

**New:** greatest integer function, piecewisedefined function, step function

## **Lesson 8.2 Graph Absolute Value Functions**– 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

• Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

#### **Mathematical Practices and Processes**

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively

#### I Can Objective

I can evaluate and graph absolute value functions.

#### **Learning Objective**

Evaluate, graph, and write absolute value functions by finding the vertex and the compression or stretch of the function.

#### Language Objective

Explain the steps needed to find an absolute value function from the vertex and the graph.

#### Vocabulary

New: vertex of an absolute-value graph, axis of symmetry



## **Lesson 8.3 Solve Absolute Value Equations and Inequalities**– 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- Explain why the x-coordinates of the points where the graphs of the equations y = f (x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/ or g (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Model with mathematics.

#### I Can Objective

I can solve absolute value equations and inequalities both graphically and algebraically.

#### **Learning Objective**

Solve absolute value equations and inequalities algebraically and by graphing

related functions, use the structure of absolute value equations and inequalities to predict the nature of solution sets, and solve absolute value inequalities that model realworld problems.

#### Language Objective

Explain the steps needed to solve an absolute value inequality.

### Vocabulary

New: tolerance



## HMH Into AGA Algebra 1

### **Unit 4: Linear Systems**

**Unit 4 Project**: STEM Task: Electrical Engineer – An Old Current Problem **Unit 4 Learning Mindset Focus**: Strategic help-seeking

### **Module 9: Systems of Linear Equations**

Recommended Pacing: 6 days

#### **Module 9 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>wrote and solved linear equations in two variables to model and solve real-world problems.</li> <li>graphed linear equations to model and solve real-world problems.</li> <li>solved or estimated solutions to systems of linear equations by graphing.</li> </ul>	<ul> <li>Students:</li> <li>solve systems of linear equations graphically, using the substitution method, using the elimination method, and by multiplying.</li> <li>use systems of linear equations to represent and solve real-world problems.</li> <li>identify when a system of linear equations has one solution, no solution, or infinitely many solutions.</li> </ul>	<ul> <li>Students:</li> <li>will graph linear inequalities and systems of linear inequalities.</li> <li>will solve nonlinear systems of equations.</li> <li>will use matrices to solve systems of equations</li> </ul>

#### **Module 9 Academic Vocabulary**

consistent system	a system that has at least one solution
dependent system	a system that has infinitely many solutions
elimination method	a method used to find the solution of a system of equations by eliminating one variable in the system by adding or subtracting the equations
inconsistent system	a system that has no solutions
independent system	a system that has exactly one solution
solution of a system of equations	any ordered pair that satisfies all the equations in a system
substitution method	a method used to find the solution of a system of equations by substituting the value of one variable in one equation in the other equation
system of equations	two or more equations having the same variables



## Lesson 9.1 Solve Linear Systems by Graphing – 1 Day

#### Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by systems of equations and interpret solutions as viable or nonviable options in a modeling context.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can write linear systems to model realworld problems and solve them by graphing

#### Learning Objective

Students will identify systems of equations by the number of solutions, solve systems of

equations by graphing, and apply systems of equations to real-world situations.

#### Language Objective

Explain the steps needed to use graphs to solve systems of linear equations.

#### Vocabulary

New: consistent system, dependent system, inconsistent system, independent system, solution of a system of equations, system of equations

Lesson Materials: graph paper

### Lesson 9.2 Solve Linear Systems by Substitution – 2 Days

#### Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

• Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Model with mathematics.

#### I Can Objective

I can write systems of linear equations to model real-world situations and solve them by substitution

#### **Learning Objective**

Write and solve systems of equations algebraically using the method of substitution. Interpret the meaning of solutions to systems, including dependent and inconsistent systems.



#### Language Objective

Explain the meaning of the solution of a system of linear equations with respect to the equations in the system and the graphs of the lines represented by the equations. Vocabulary Review: substitution method

## **Lesson 9.3 Solve Linear Systems by Adding or Subtracting**– 1 Day Focus on:

Apply and Practice

#### **Mathematics Standards**

- Solve systems of linear equations exactly . . ., focusing on pairs of linear equations in two variables.
- Create equations in two or more variables to represent relationships between quantities
- Represent constraints by equations . . ., and by systems of equations . . ., and interpret solutions as viable or nonviable options in a modeling context

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Model with mathematics.

#### I Can Objective

I can write linear systems to model realworld situations and solve the systems by adding or subtracting.

#### **Learning Objective**

Students will write systems of linear equations to model real-world situations and solve them by adding or subtracting.

#### Language Objective

Explain the steps needed to solve a system of linear equations by adding or subtracting.

#### Vocabulary

New: elimination method

## **Lesson 9.4 Solve Linear Systems by Multiplying First** – 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### **Mathematical Practices and Processes**

#### I Can Objective

- Look for and make use of structure.
- Model with mathematics.

I can write linear systems to model realworld problems and solve them by multiplying first.



#### Learning Objective

Write systems of linear equations to model real-world situations and solve them by multiplying first.

#### Language Objective

Explain the steps needed to solve an independent system of linear equations by multiplying first.



## Module 10: Linear Inequalities Recommended Pacing: 4 days

#### Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>wrote, solved, and graphed linear inequalities in one variable.</li> <li>used inequalities in one variable to model real-world problems.</li> <li>wrote, solved, and graphed systems of linear equations.</li> <li>modeled real-world problems with systems of equations.</li> </ul>	<ul> <li>Students:</li> <li>write, solve, and graph linear inequalities in two variables.</li> <li>write and graph a linear inequality to model and solve a real-world problem.</li> <li>write, graph, and solve systems of linear inequalities.</li> <li>model real-world problems with systems of linear inequalities.</li> </ul>	Students: • will solve nonlinear systems of equations and inequalities.

#### Module 10 Academic Vocabulary

boundary line	the line that divides the coordinate plane into two half-planes
half-plane	part of the coordinate plane on one side of a line, which may include the line
linear inequality in two variables	an inequality that can be written in one of the following forms: $Ax + By < C$ , $Ax + By > C$ , $Ax + By \le C$ , $Ax + By \ge C$ , or $Ax + By \ne C$ , where A, B, and C are real numbers and A and B are not both 0.
solution of an inequality in two variables	any ordered pair that makes the inequality true
solution of a system of linear inequalities	any ordered pair that satisfies all inequalities in a system of inequalities
system of linear inequalities	a system of two or more linear inequalities having the same variables



## Lesson 10.1 Linear Inequalities in Two Variables – 2 Days

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Attend to precision.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can write and graph linear inequalities in two variables to model and solve real-world problems

#### **Learning Objective**

Write and graph linear inequalities in two variables to model and solve real-world problems

#### Language Objective

Describe the process for graphing a linear inequality in two variables.

**Vocabulary:** New: boundary line, half-plane, linear inequality in two variables, solution of an inequality in two variables

## **Lesson 10.2 Arithmetic Sequences Defined Explicitly** – 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively
- Model with mathematics.

#### I Can Objective

I can write and graph systems of linear inequalities to model and solve real-world problems

#### **Learning Objective**

Write and graph systems of linear inequalities to model and solve real-world problems

#### Language Objective

Explain the steps needed to graph a system of linear inequalities

Vocabulary: New: solution of a system of linear inequalities, system of linear inequalities



## HMH Into AGA Algebra 1

### **Unit 5: Linear Systems**

**Unit 5 Project**: STEM Task: Industrial Engineer – Oh, How We Grow! **Unit 5 Learning Mindset Focus**: Perseverance: Checks for Understanding

### **Module 11: Exponential Functions and Models**

Recommended Pacing: 5 days

#### **Module 11 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>analyzed graphs to describe relationships.</li> <li>applied the properties of integer exponents.</li> <li>worked with rational exponents.</li> <li>modeled with linear functions.</li> </ul>	<ul> <li>Students:</li> <li>analyze characteristics of exponential functions.</li> <li>graph exponential functions.</li> <li>model with exponential functions.</li> <li>write exponential models to find doubling rate or half-life</li> </ul>	<ul> <li>Students:</li> <li>will transform exponential functions.</li> <li>will compare exponential functions.</li> <li>will fit exponential functions to data.</li> <li>will choose between models, including exponential ones.</li> <li>will use discrete exponential functions.</li> </ul>

#### Module 11 Academic Vocabulary

acumptoto	a line that a graph gets closer to as the value of x increases or decreases
asymptote	without bound
exponential decay	a function of the form f( x) = ab x , where a and b are real numbers, a > 0, and 0
function	< b < 1
exponential	a function of the form $f(x) = ab x$ , where a and b are real numbers, $a \neq 0$ , and b
function	> 0 and ≠1
exponential growth	a function of the form f( x) = ab x , where a and b are real numbers, a > 0, and b
function	>1



## **Lesson 11.1 Exponential Growth Functions** – 2 Days Focus on:

#### Build Conceptual Understanding

#### **Mathematics Standards**

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Use appropriate tools strategically.
- Model with mathematics.

#### I Can Objective

I can write, graph, and analyze exponential growth functions.

#### Learning Objective

Write, graph, and analyze exponential models.

#### Language Objective

Explain the characteristics of an exponential growth function.

#### Vocabulary

New: asymptote, exponential function, exponential growth function

## **Lesson 11.2 Exponential Decay Functions**- 2 Days Focus on:

#### Build Conceptual Understanding

#### **Mathematics Standards**

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Use appropriate tools strategically.
- Model with mathematics.

#### I Can Objective

I can analyze the characteristics of exponential decay functions.

#### Learning Objective

Use properties of exponents and graphing to analyze exponential growth and decay models.

#### Language Objective

Explain the characteristics of an exponential decay function. **Vocabulary** New: exponential decay function



## **Lesson 11.3 Rewrite Exponential Models** – 1 Day Focus on:

Apply and Practice

#### **Mathematics Standards**

- Use the properties of exponents to transform expressions for exponential functions.
- Use the properties of exponents to interpret expressions for exponential functions.
- Interpret parts of an expression, such as terms, factors, and coefficients.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Model with mathematics.

#### I Can Objective

I can use the properties of exponents to write exponential models.

#### **Learning Objective**

Use properties of exponents and graphing to construct exponential growth and decay models.

#### Language Objective

Explain how to convert the base of an exponential model to give the doubling rate or half-life



# Module 12: Relationships Among Exponential Functions Recommended Pacing: 4 days

#### Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>transformed functions.</li> <li>wrote exponential growth and decay functions.</li> <li>evaluated exponential growth and decay functions.</li> <li>solved problems involving rates.</li> </ul>	<ul> <li>Students:</li> <li>describe the effects of transformations on the characteristics of exponential functions.</li> <li>transform a transformed exponential function.</li> <li>model real-world problems with transformed exponential functions.</li> <li>express different forms of exponential functions.</li> <li>compare growth and decay rates of exponential functions.</li> </ul>	<ul> <li>Students:</li> <li>will fit an exponential curve to a data set.</li> <li>will choose between linear and exponential models.</li> </ul>



## **Lesson 12.1 Transform Exponential Functions**- 2 Days Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

• Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can graph and identify transformations of exponential functions.

#### **Learning Objective**

Interpret exponential functions in the form f(x) = a(bx) + k and determine how

## changing a, k, or both can transform the exponential parent function.

#### Language Objective

Communicate the relationship between a exponential parent function and its transformations.

#### Vocabulary

New: boundary line, half-plane, linear inequality in two variables, solution of an inequality in two variables

## **Lesson Materials** graph paper

## **Lesson 12.2 Compare Exponential Functions** – 2 Days Focus on:

### Apply and Practice

#### Mathematics Standards

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Explain why the x-coordinates of the points where the graphs of the equations y = f (x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively
- Model with mathematics.
- Look for and express regularity in repeated reasoning.
- Construct viable arguments and critique the reasoning of others.

#### I Can Objective

I can compare exponential functions given in different forms

#### Learning Objective

Evaluate different expressions of a function and compare growth and decay rates and function values, including initial values

#### Language Objective

Communicate by speaking or writing a comparison involving one or more exponential functions.



## HMH Into AGA Algebra 1

### **Unit 6: Build Exponential Functions and Models**

**Unit 6 Project**: STEM Task: Aeronautical Engineer – What a Drag! **Unit 6 Learning Mindset Focus**: Strategic help-seeking

### **Module 13: Fit Exponential Functions to Data**

Recommended Pacing: 4 days

#### **Module 13 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>modeled with linear functions.</li> <li>analyzed exponential growth and decay models.</li> <li>compared exponential functions in different forms.</li> <li>fit linear functions to data.</li> <li>fit exponential curves to data.</li> </ul>	<ul> <li>Students:</li> <li>model exponential data by hand and with technology.</li> <li>use residuals to determine the quality of fit of a model.</li> <li>model exponential data using a piecewise-defined functions.</li> <li>compare linear and exponential relationships.</li> <li>choose between linear and exponential models.</li> </ul>	Students: • will use exponential functions to model geometric sequences.

#### Module 13 Academic Vocabulary

**exponential** a graphing calculator tool used to generate an exponential equation that fits exponential growth or decay data



### **Lesson 13.1 Scatter Plots and Fitted Exponential Curves** – 2 Days Focus on:

Build Conceptual Understanding

#### Mathematics Standards

- Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
- Informally assess the fit of a function by plotting and analyzing residuals.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

#### I Can Objective

I can fit exponential functions to data and make predictions about real-world situations.

#### **Learning Objective**

Fit exponential functions to data and make predictions for real-world data.

#### Language Objective

Describe the process for using exponential models to make predictions.

**Vocabulary** New: exponential regression **Lesson Materials** graphing calculator

## **Lesson 13.2 Solve Linear Systems by Substitution** – 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

#### I Can Objective

I can choose between linear and exponential models for given data sets.

#### Learning Objective

Choose between linear and exponential models for given data sets.



**Language Objective** Describe the process for determining whether a linear or exponential function best models a given data set.

**Lesson Materials** graphing calculator



# Module 14: Discrete Exponential Functions Recommended Pacing: 4 days

#### **Module 14 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>wrote exponential growth models.</li> <li>wrote exponential decay models.</li> <li>defined arithmetic sequences recursively.</li> <li>defined arithmetic sequences explicitly.</li> </ul>	<ul> <li>Students:</li> <li>define and identify geometric sequences.</li> <li>write and evaluate a recursive formula for a geometric sequence.</li> <li>recognize the graph of a geometric sequence.</li> <li>express geometric sequences explicitly.</li> <li>apply geometric sequences to real-world problems.</li> <li>convert between explicit and recursive</li> <li>rules for a geometric sequence.</li> </ul>	<ul> <li>Students:</li> <li>will choose among linear, exponential, and quadratic models.</li> <li>will work with arithmetic and geometric series</li> </ul>

### Module 14 Academic Vocabulary

common ratio	the constant ratio of successive terms in a geometric sequence
geometric sequence	a sequence where the ratio of successive terms is a constant, called the common ratio



## **Lesson 14.1 Geometric Sequences Defined Recursively**– 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Attend to precision.
- Model with mathematics.

#### I Can Objective

I can write recursive formulas for geometric sequences.

#### **Learning Objective**

Write recursive formulas for geometric sequences.

#### Language Objective

Describe the process for writing a recursive rule for a geometric sequence.

#### Vocabulary

New: common ratio, geometric sequence

### Lesson 14.2 Geometric Sequences Defined Explicitly- 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

#### **Mathematical Practices and Processes**

- Attend to precision.
- Look for and make use of structure.
- Reason abstractly and quantitatively
- Model with mathematics.

#### I Can Objective

I can write explicit rules for geometric sequences

#### **Learning Objective**

Write explicit rules for geometric sequences

#### Language Objective

Describe the process for converting between explicit and recursive rules for geometric sequences.



## HMH Into AGA Algebra 1

### **Unit 7: Polynomial Operations and Models**

**Unit 7 Project**: STEM Task: Computer Scientist – Breaking a Hex **Unit 7 Learning Mindset Focus**: Resilience: Manages the Learning Process

### **Module 15: Polynomial Multiplication**

Recommended Pacing: 6 days

#### **Module 15 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>used the Distributive Property.</li> <li>applied properties of exponents.</li> <li>wrote and simplified algebraic expressions.</li> <li>wrote and solved linear equations.</li> </ul>	<ul> <li>Students:</li> <li>recognize characteristics of monomials.</li> <li>multiply monomials by other monomials.</li> <li>recognize characteristics of polynomial expressions, including monomials, binomials, and trinomials.</li> <li>multiply polynomial expressions.</li> <li>model area and volume with polynomial multiplication.</li> <li>solve problems using patterns in special products of binomials.</li> </ul>	<ul> <li>Students:</li> <li>will add and subtract polynomials</li> <li>will model with polynomials.</li> <li>will solve quadratic equations by factoring.</li> <li>will use special factoring patterns to solve quadratic equations.</li> </ul>

#### Module 15 Academic Vocabulary

binomial	a polynomial that has two terms
degree of a monomial	the sum of the exponents of the variables in the monomial
degree of a polynomial	the degree of the term of the polynomial with the greatest degree
leading coefficient	the coefficient of the first term of a polynomial written in standard form
monomial	a number, a variable, or a product of numbers and variables with whole- number exponents
perfect-square trinomial	a trinomial that is the result of squaring a binomial.
polynomial	a monomial or a sum of monomials
standard form of a polynomial	the form in which a polynomial in one variable is written when the terms are in order from greatest degree to least degree
trinomial	a polynomial that has three terms



### Lesson 15.1 Multiple Monomials – 2 Days

#### Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Understand that polynomials form a system analogous to the integers, namely, they are closed under operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Look for and make use of structure.

#### I Can Objective

I can multiply monomials and raise monomials to powers to solve real-world problems.

#### **Learning Objective**

Multiply monomials and raise monomials to powers to solve real-world problems.

### Language Objective

Explain the characteristics of monomials.

#### **Vocabulary New:** degree of a monomial, monomial

Lesson Materials: algebra tiles

### Lesson 15.2 Multiply Monomials, Binomials, and Trinomials– 2 Days Focus on:

#### Connect Concepts and Skills

#### **Mathematics Standards**

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Look for and make use of structure.

#### I Can Objective

I can write monomials, binomials, and trinomials in standard form and multiply them.

#### **Learning Objective**

Multiply monomials, binomials, and trinomials.

#### Language Objective

Explain how to multiply two binomials

#### Vocabulary

New: binomial, degree of a polynomial, leading coefficient, polynomial, standard form of a polynomial, trinomial



## **Lesson 15.3 Special Products of Binomials** – 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

- Solve systems of linear equations exactly . . ., focusing on pairs of linear equations in two variables.
- Understand that polynomials form a system analogous to the integers. Namely, they are closed under operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Use the structure of an expression to identify ways to rewrite it.

#### **Mathematical Practices and Processes**

- Look for and express regularity in repeated reasoning.
- Look for and make use of structure.
- Model with mathematics.

#### I Can Objective

I can find special products of binomials.

#### **Learning Objective**

Find special products of binomials.

#### Language Objective

Explain the patterns found in certain types of binomial multiplication.

#### Vocabulary

New: perfect-square trinomial



## Module 16: Polynomial Addition and Subtraction

Recommended Pacing: 3 days

#### **Module 16 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>multiplied monomial expressions in the context of a real-world problem.</li> <li>defined binomial, trinomial, polynomial, degree, and leading coefficient.</li> <li>multiplied polynomials.</li> <li>calculated the area of a figure with polynomial dimensions.</li> </ul>	<ul> <li>Students:</li> <li>distinguish between like and unlike terms.</li> <li>apply the Distributive Property to subtract polynomials.</li> <li>add and subtract polynomials.</li> <li>use polynomials to represent figurate numbers.</li> <li>model real-world situations with polynomials.</li> </ul>	<ul> <li>Students:</li> <li>will factor polynomials.</li> <li>will solve quadratic equations by factoring</li> </ul>

#### Module 16 Academic Vocabulary

**figurate numbers** numbers that can be represented by evenly spaced dots arranged in a geometric shape



## Lesson 16.1 Add and Subtract Polynomials – 2 Days

#### Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

#### I Can Objective

I can add and subtract polynomials.

#### **Learning Objective**

Identify like terms in polynomials in the process of addition or subtraction.

#### Language Objective

Explain how adding and subtracting polynomials requires identifying like and unlike terms.

### Lesson 16.2 Model with Polynomials – 1 Day

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Understand that polynomials form a system analogous to the integers. Namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Determine an explicit expression, a recursive process, or steps for calculation from a context.

#### **Mathematical Practices and Processes**

- Construct viable arguments and critique the reasoning of others
- Model with mathematics.

#### I Can Objective

I can model mathematical and real-world problems with polynomials

#### **Learning Objective**

Model mathematical and real-world problems with polynomials

#### Language Objective

Explain how to use polynomials to model real-world problems.

**Vocabulary** New: figurate numbers



## HMH Into AGA Algebra 1

### **Unit 8: Quadratic Functions and Equations**

**Unit 8 Project**: STEM Task: Mechanical Engineer – Around and Around **Unit 8 Learning Mindset Focus**: Challenge-seeking: Makes Plans to Meet Goals

## Module 17: Use Graphing and Factoring to Solve Quadratic Equations

Recommended Pacing: 8 days

#### **Module 17 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>graphed linear equations.</li> <li>wrote and solved linear equations.</li> </ul>	<ul> <li>Students:</li> <li>graph quadratic equations and identify characteristics of the graphs.</li> <li>solve quadratic equations graphically, by factoring, and by using the Zero Product Property.</li> <li>factor a quadratic expression to reveal the zeros of the function it defines.</li> <li>create quadratic equations in one variable and use them to solve problems.</li> </ul>	<ul> <li>Students:</li> <li>will solve quadratic equations by methods other than graphing.</li> <li>will choose an appropriate method for solving quadratic equations.</li> <li>will take square roots to solve quadratic equations.</li> <li>will complete the square to solve quadratic equations.</li> <li>will use the Quadratic Formula to solve equations.</li> </ul>

#### Module 17 Academic Vocabulary

parabola	the graph of a quadratic function
quadratic function	can be written in the form f(x) = a x 2 + bx + c, where a, b, and c are real numbers and a $\neq 0$
standard form	a quadratic equation written in the form a x $2 + bx + c = 0$
vertex of a parabola	the turning point given by the highest or lowest point on a parabola
zero	any number x such that $f(x) = 0$



### Lesson 17.1 Solve Quadratic Equations by Graphing Quadratic Functions – 2 Days

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Create equations and inequalities in one variable and use them to solve problems.

#### **Mathematical Practices and Processes**

- Model with mathematics.
- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.
- Construct viable arguments and critique the reasoning of others.

#### I Can Objective

I can understand the key features of the graph of a quadratic function, and solve quadratic equations approximately by graphing.

#### **Learning Objective**

Graph quadratic equations, identify the key features of the graphs, and use them to solve related equations.

#### Language Objective

Communicate relationships among a quadratic equation, its graph, and solutions to the related equation.

#### Vocabulary

New: quadratic function, parabola, vertex of a parabola, zero

#### **Lesson Materials**

graph paper, graphing calculator

## Lesson 17.2 Solve Quadratic Equations by Factoring x 2 + bx + c -

### 2 Days

Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Solve quadratic equations by inspection (e.g., for x 2 = 49), using square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a  $\pm$  bi for real numbers a and b.
- Use the structure of an expression to identify ways to rewrite it.
- Create equations and inequalities in one variable and use them to solve problems.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
- Model with mathematics.

#### I Can Objective

I can use the Zero Product Property to solve quadratic equations in standard form with a leading coefficient of 1.



#### **Learning Objective**

Solve quadratic equations in the form of x 2 + bx + c = 0 by factoring and using the Zero Product Property. Explain the steps followed to solve a quadratic equation using the Zero Product Property.

#### Language Objective

**Vocabulary** New: standard form

### Lesson 17.3 Solve Quadratic Equations by Factoring ax 2 + bx + c- 2

#### Days

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a  $\pm$  bi for real numbers a and b.
- Use the structure of an expression to identify ways to rewrite it.
- Create equations and inequalities in one variable and use them to solve problems.

#### **Mathematical Practices and Processes**

- Model with mathematics.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively

#### I Can Objective

I can use the Zero Product Property to solve quadratic equations in standard form when the leading coefficient is not 1.

#### **Learning Objective**

Use the Zero Product Property to solve quadratic equations in standard form when the leading coefficient is not 1.

#### Language Objective

Explain the steps needed to factor expressions of the form a x + bx + c, and use this factored form to solve quadratic equations.

Lesson Materials: spreadsheet software

## Lesson 17.4 Use Special Factoring to Solve Quadratic Equations- 2

#### Days

Focus on:

Apply and Practice

#### **Mathematics Standards**

- Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots ,completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
- Use the structure of an expression to identify ways to rewrite it.



• Create equations and inequalities in one variable and use them to solve problem.

#### **Mathematical Practices and Processes**

- Model with mathematics.
- Look for and express regularity in repeated reasoning.
- Reason abstractly and quantitatively

#### I Can Objective

I can use special factoring patterns to solve quadratic equations.

#### Learning Objective

Use special factoring patterns to solve quadratic equations.

#### Language Objective

Explain the terms difference of squares and perfect square trinomial factoring to recognize and explain the special patterns used to factor in this lesson.



## Module 18: Use Square Roots to Solve Quadratic Equations

Recommended Pacing: 7 days

#### **Module 18 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>solved linear equations.</li> <li>solved quadratic equations by graphing.</li> <li>solved quadratic equations by factoring.</li> </ul>	<ul> <li>Students:</li> <li>use square roots to solve quadratic equations.</li> <li>solve equations by completing the square.</li> <li>solve quadratic equations using the Quadratic Formula.</li> <li>choose an appropriate method to solve a quadratic equation.</li> </ul>	<ul> <li>Students:</li> <li>will write quadratic functions in vertex form, standard form, and intercept form.</li> <li>will compare quadratic functions and models.</li> </ul>

#### Module 18 Academic Vocabulary

completing the square	a process used to form a perfect-square trinomial. To complete the square for x $2 + bx$ , add $(b/2)2$
discriminant	The discriminant of the quadratic equation $a \ge 2 + b \ge -2 + b \ge 2 - 4ac$ .
Quadratic Formula	The formula x = -b $\pm \sqrt{b^2}$ - 4ac/2a , which gives solutions of equations in the form a x 2 + bx + c = 0, where a $\neq$ 0.



## **Lesson 18.1 Solve Simple Quadratic Equations** – 2 Days Focus on:

Build Conceptual Understanding

#### **Mathematics Standards**

- Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a  $\pm$  bi for real numbers a and b.
- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can solve quadratic equations of the form a x = c by using square roots and determine whether there are one, two, or no real solutions.

#### **Learning Objective**

Solve quadratic equations of the form a x 2 = c and a(x + b) 2 = c by using square roots and determine whether there are one, two, or no real solutions.

#### Language Objective

Describe the process for solving quadratic equations by using square roots.

#### **Lesson Materials**

graph paper and graphing calculators

## Lesson 18.2 Solve Quadratic Equations by Completing the Square –

### 2 Days

Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p) 2 = q that has the same solutions. Derive them quadratic formula from this form.
- Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a  $\pm$  bi for real numbers a and b.

#### **Mathematical Practices and Processes**

#### I Can Objective

- Use tools.
- Reason abstractly and quantitatively.
- Look for and make use of structure

I can solve quadratic equations of the form a x + 2 + bx + c = 0 by completing the square.



#### **Learning Objective**

Solve mathematical and real-world quadratic equations of the form  $a \ge 2 + b \ge + c$ = 0 by completing the square.

**Language Objective** By verbal and written communication, students should be able to relate procedures and reasoning used to solve quadratic equations by completing the square.

**Vocabulary** New: completing the square

Lesson Materials algebra tiles

### Lesson 18.3 Use the Quadratic Formula to Solve Equations – 2 Days

Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p) 2 = q that has the same solutions. Derive the quadratic formula from this form.

#### **Mathematical Practices and Processes**

- Model with mathematics.
- Look for and make use of structure.

#### **Learning Objective**

Use the method of completing the square to derive the Quadratic formula, and solve equations by using the Quadratic Formula.

#### I Can Objective

I can use the method of completing the square to derive the Quadratic Formula and solve equations by using the Quadratic Formula

#### Language Objective

Describe the process for solving quadratic equations using the Quadratic Formula.

#### Vocabulary

New: discriminant, Quadratic Formula

# **Lesson 18.4 Choose a Method for Solving Quadratic Equations –** 1 Day

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

- Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a  $\pm$  bi for real numbers a and b.
- Create equations and inequalities in one variable and use them to solve problems.



#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Model with mathematics.

#### I Can Objective

I can solve quadratic equations using a variety of methods and choose an appropriate method based on the initial form of the equation.

#### **Learning Objective**

Solve quadratic equations by any method and choose an appropriate method based on the initial form of the equation.

#### Language Objective

Explain the reasoning behind your choice of which method to use when solving a quadratic equation.



## HMH Into AGA Algebra 1

### **Unit 9: Functions and Models**

**Unit 9 Project**: STEM Task: Demographer –Elementary My Dear, Demographer **Unit 9 Learning Mindset Focus**: Perseverance: Collects and Tries Multiple Strategies

### **Module 19: Build Quadratic Functions and Models**

Recommended Pacing: 10 days

#### **Module 19 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>wrote and solved linear equations in two variables to model and solve real-world problems.</li> <li>wrote the standard form of a quadratic equation.</li> <li>determined the number that completed the square.</li> <li>identified the zeros of a function.</li> <li>determined the number of real solutions of a quadratic equation.</li> <li>solved quadratic equations by graphing quadratic functions.</li> <li>solved quadratic equations by completing the square.</li> <li>fit linear functions to data.</li> </ul>	<ul> <li>Students:</li> <li>graph a quadratic function using transformations.</li> <li>identify the vertex, maximum, minimum, reflection, and stretch for a parabola from the graph or from the equation.</li> <li>convert between standard, vertex, and intercept forms.</li> <li>compare quadratic functions represented in different ways.</li> <li>compute second differences to determine if data is quadratic.</li> <li>fit quadratic curves to data by hand and using regression.</li> </ul>	<ul> <li>Students:</li> <li>will choose among linear, exponential, and quadratic models.</li> <li>will graph cubic functions.</li> <li>will identify the zeros and end behavior of a cubic function.</li> <li>will graph polynomial functions.</li> <li>will solve polynomial functions.</li> </ul>

#### Module 19 Academic Vocabulary

coefficient of determination	a statistical value from 0 to 1 that is denoted by R 2 which tells you how well a function fits a set of data
first differences	the differences between the y-values of a function for evenly spaced x-values
intercept form of a quadratic function	the form f ( x) = a( x – x 1 ) ( x – x 2 ) , where x 1 and x 2 are the zeros of f( x)
quadratic regression	a statistical method used to fit a quadratic model to a given data set
second differences	the differences between first differences
vertex form of a quadratic function	the form $f(x) = a(x - h) 2 + k$ of a quadratic function, where a, h, and k are constants



## **Lesson 19.1 Quadratic Functions in Vertex Form**– 2 Days Focus on:

Build Conceptual Understanding

#### Mathematics Standards

- Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

#### **Mathematical Practices and Processes**

- Attend to precision.
- Look for and make use of structure.

#### I Can Objective

I can write and graph quadratic functions in vertex form.

#### **Learning Objective**

Deliberately manipulate the terms of a quadratic function in vertex form to perform

transformations on the parent function, and interpret a parabola to write its function.

#### Language Objective

Explain how to determine the transformations on the parent quadratic function given the equation or the graph.

#### **Vocabulary** New: vertex form of a quadratic function

## Lesson 19.2 Quadratic Functions in Standard Form – 2 Days

Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

#### I Can Objective

I can graph quadratic functions in standard form and convert between standard and vertex forms of quadratic functions.



#### **Learning Objective**

Graph quadratic functions in standard form and convert between standard and vertex forms of quadratic functions.

#### Language Objective

Explain how the standard and vertex forms of a quadratic function are related.

#### **Lesson Materials** prepared index cards, graph paper

## **Lesson 19.3 Quadratic Functions in Intercept Form** – 2 Days Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

- Solve systems of linear equations exactly . . ., focusing on pairs of linear equations in two variables.
- Factor a quadratic expression to reveal the zeros of the function it defines.
- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- Graph linear and quadratic functions and show intercepts, maxima, and minima

#### **Mathematical Practices and Processes**

- Look for and express regularity in repeated reasoning.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

#### I Can Objective

I can write and graph quadratic functions in intercept form.

#### Learning Objective

Use a quadratic function in intercept form to graph a parabola.

#### Language Objective

Explain how to get the key features for graphing a parabola from the intercept form of the quadratic function.

#### Vocabulary

New: intercept form of a quadratic function

#### **Lesson Materials**

index cards, graph paper

## **Lesson 19.4 Compare Quadratic Functions** – 2 Days

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

• Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).



• Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

#### **Mathematical Practices and Processes**

• Look for and make use of structure.

#### I Can Objective

I can compare quadratic functions represented in different forms.

#### **Learning Objective**

Compare quadratic functions represented in different forms.

#### Language Objective

Describe the process for comparing pairs of quadratic functions represented in different ways.

### Lesson 19.5 Scatter Plots and Fitted Quadratic Curves– 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.

#### I Can Objective

I can fit quadratic functions to data by hand and by quadratic regression.

#### **Learning Objective**

Fit quadratic functions to data by hand and by quadratic regression.

#### Language Objective

Describe the process for fitting a quadratic curve to data by hand.

#### Vocabulary

New: coefficient of determination, first differences, quadratic regression, second differences

#### **Lesson Materials** graphing calculator

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## Module 20: Function Analysis Recommended Pacing: 8 days

#### Module 20 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>determined the best for data.</li> <li>added, subtracted, multiplied, and divided polynomials.</li> <li>wrote quadratic functions in standard form and intercept form.</li> </ul>	<ul> <li>Students:</li> <li>use regression and residual plots to choose models for data.</li> <li>add, subtract, multiply, and divide models.</li> <li>solve nonlinear systems graphically, using tables, and by successive approximations.</li> <li>analyze, write, and graph cubic functions</li> </ul>	<ul> <li>Students:</li> <li>will graph polynomial functions.</li> <li>will perform operations with trigonometric functions.</li> <li>will solve nonlinear systems algebraically.</li> <li>will factor polynomials.</li> <li>will find inverses of cubic functions.</li> </ul>

#### Module 20 Academic Vocabulary

cubic function	the a function defined by a third-degree polynomial expression
standard form of a cubic function	f(x) = a x3 + b x2 + cx + d, where a, b, c, and d are real numbers and a $\neq 0$



### Lesson 20.1 Choose Among Linear, Quadratic, and Exponential Models – 2 Days

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

- Fit a function to data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.

#### I Can Objective

I can determine whether a given data set is best modeled by a linear, exponential, or quadratic function.

#### **Learning Objective**

Determine whether a given data set is best modeled by a linear, exponential, or quadratic function

#### Language Objective

Based on patterns observed in data sets, explain what type of function model would best represent the data.

#### **Lesson Materials**

graphing calculator or other software that can perform linear, quadratic, and exponential regression on pairs of points



## **Lesson 20.2 Perform Operations with Functions** – 2 Days Focus on:

Apply and Practice

#### **Mathematics Standards**

• Combine standard function types using arithmetic operations.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Model with mathematics.

#### **Learning Objective**

Perform arithmetic operations with functions in mathematical and real-world contexts

#### I Can Objective

I can perform operations with functions in mathematical and real-world contexts

#### Language Objective

Explain how to perform operations on functions

### Lesson 20.3 Solve Nonlinear Systems – 2 Days

Focus on:

Apply and Practice

#### **Mathematics Standards**

- Explain why the x-coordinates of the points where the graphs of the equations y = f(x)and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Attend to precision.

#### I Can Objective

I can use graphs, tables, and successive approximations to solve nonlinear systems involving linear, quadratic, and exponential equations

#### **Learning Objective**

Use graphs, tables, and successive approximations to solve nonlinear systems involving linear, quadratic, and exponential equations.

#### Language Objective

Describe the process for comparing pairs of quadratic functions represented in different ways.

#### **Lesson Materials**

graphing calculator or other graphing software



### Lesson 20.4 Cubic Functions – 2 Days

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

- Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

#### **Mathematical Practices and Processes**

- Look for and make use of structure.
- Use appropriate tools strategically.
- Model with mathematics.

#### I Can Objective

I can analyze cubic functions and their behaviors

#### **Learning Objective**

Analyze cubic functions and the behavior of cubic functions by graphing them to model and solve real-world problems.

#### Language Objective

Explain the steps needed to analyze cubic functions, graph cubic functions, and identify zeros and intercepts.

#### Vocabulary

New: cubic function, standard form of a cubic function

#### **Lesson Materials**

graphing calculator or graphing software



## HMH Into AGA Algebra 1

### **Unit 10: Data Analysis**

**Unit 10 Project**: STEM Task: Ecologist – Ecology Is for the Birds **Unit 10 Learning Mindset Focus**: Resilience: Notices Others

### Module 21: Categorical Data

Recommended Pacing: 4 days

#### **Module 21 Mathematical Progressions**

Prior Learning	Current Development	Future Connections
Students: • constructed and interpreted two-way frequency tables and two-way relative frequency tables.	<ul> <li>Students:</li> <li>summarize categorical data for two categories in two- way frequency tables.</li> <li>interpret relative frequencies in context.</li> <li>recognize possible associations between categorical variables.</li> </ul>	<ul> <li>Students:</li> <li>will calculate probabilities from two-way tables.</li> <li>will identify possible associations in numerical data.</li> <li>will distinguish between correlation and causation.</li> </ul>

#### Module 21 Academic Vocabulary

conditional relative	the ratio of a joint relative frequency to a related marginal relative frequency in
frequency	a two-way table
frequency	indicates the number of items in a data set that belong to both a specific category for one of the categorical variables and another specific category for the other categorical variable
joint relative	a relative frequency representing a combination of one category of the
frequency	categorical variables and another category from the other categorical variable
marginal relative frequency	the sum of the joint relative frequencies in a row or column of a two-way table
relative frequency	the quotient of any frequency in a two-way frequency table and the total number of data values
two-way frequency table	shows frequency data for two categorical variable
two-way relative frequency table	a two-way frequency table that displays relative frequencies
categorical data	qualitative data
quantitative data	numerical data



## Lesson 21.1 Two-Way Frequency and Relative Frequency Tables – 2

#### Days

#### Focus on:

Connect Concepts and Skills

#### **Mathematics Standards**

• Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.
- Construct viable arguments and critique the reasoning of others.

#### I Can Objective

I can read, create, and interpret two-way frequency tables and relative frequency tables.

#### **Learning Objective**

Read, create, and interpret a two-way frequency table and a relative frequency table for data collected through a survey.

#### Language Objective

Explain the steps needed to create a relative frequency table from data collected and displayed in a two-way frequency table.

#### Vocabulary

Review: frequency, joint relative frequency, marginal relative frequency, relative frequency, two-way frequency table, two way relative frequency table New: categorical data, quantitative data

### Lesson 21.2 Recognize Possible Associations Between Categorical Variables – 2 Days

#### Focus on:

Apply and Practice

#### **Mathematics Standards**

• Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

#### I Can Objective

I can calculate conditional relative frequencies and use them to identify

possible associations between categorical variables.

#### **Learning Objective**

Calculate conditional relative frequencies and use them to identify possible associations between categorical data.



#### Language Objective

Explain how to use the calculated relative frequencies to outline an association that exists in collected data.

#### Vocabulary

Review: conditional relative frequency

Lesson Materials: index cards



## Module 22: Numerical Data

Recommended Pacing: 4 days

#### Module 22 Mathematical Progressions

Prior Learning	Current Development	Future Connections
<ul> <li>Students:</li> <li>Found measures of center and variability of a data set, such as mean, median, and range.</li> <li>read, created, and interpreted two-way frequency tables and relative frequency tables.</li> </ul>	<ul> <li>Students:</li> <li>understand and analyze measures of center and spread.</li> <li>understand and identify outliers.</li> <li>create dot plots and histograms to classify distribution shapes and choose appropriate measures of center and spread.</li> <li>use statistics to compare data sets.</li> </ul>	<ul> <li>Students:</li> <li>will collect and analyze experimental data.</li> <li>will compare data using z-scores.</li> </ul>

### Module 22 Academic Vocabulary

box plot	a statistical graph that shows how values in a data set are distributed and five key values that summarize the data
histogram	statistical graph used to display data grouped into intervals of equal width
mean	the sum of all the values in a data set divided by the number of data values
median	the middle value an ordered data set with an odd number of values, or the average of the two middle values
dot plot	a number line with marks or dots to show frequency
interquartile range (IQR)	the difference of the third (upper) and first (lower) quartiles in a data set
outlier	a data value that is far removed from the rest of the data
standard deviation	a measure of dispersion of a data set; the standard deviation is the square root of the variance
statistic	a number that characterizes a data set



## **Lesson 22.1 Data Distributions and Appropriate Statistics** – 2 Days Focus on:

#### Connect Concepts and Skills

#### **Mathematics Standards**

- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- Represent data with plots on the real number line (dot plots, histograms, and box plots).

#### **Mathematical Practices and Processes**

- Attend to precision.
- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

#### I Can Objective

I can identify distribution shapes and outliers and use them to choose appropriate measures of center and spread.

**Learning Objective** Identify distribution shapes and outliers and use them to choose

appropriate measures of center and spread for describing data sets.

#### Language Objective

Describe the process for determining the interquartile range of a data set.

#### Vocabulary

Review: mean, median New: dot plot, interquartile range (IQR), outlier, standard deviation, statistic

## **Lesson 22.2 Compare Data Distributions** – 2 Days Focus on:

### Apply and Practice

#### **Mathematics Standards**

- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- Represent data with plots on the real number line (dot plots, histograms, and box plots).

#### **Mathematical Practices and Processes**

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically

#### I Can Objective

I can compare data distributions using box plots and histograms.

#### **Learning Objective**

Compare data distributions using box plots and histograms, and determine which statistical graphs are most useful for a given comparison.

#### Language Objective

Explain how to display data using histograms and box plots so that useful



comparisons can be made between the data sets.

**Vocabulary** Review: box plot, histogram Lesson Materials: calculator

