HMH into Math™ Advanced 2

Unit 1: Transform and Construct Geometric Figures

Unit 1 Project: A Puzzling Transformation
Unit 1 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 1: Transformations and Congruence

Recommended Pacing with Assessments: 12 Days

Module 1 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and drew geometric shapes with given conditions. Students used a ruler to measure length. Students used a protractor to measure angles. Students drew figures in the coordinate plane.	Students explore and verify the properties of lines and angles in transformations. Students use coordinates to describe the effect of translations, reflections, and rotations. Students execute a sequence of transformations on a figure in the coordinate plane that results in a congruent figure.	Students will represent, compare, and recognize congruent figures using transformations. Students will understand and use translations, reflections, and rotations.

Module 1 Vocabulary

coordinate plane	a plane formed by the intersection of a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis
corresponding angles	for two lines intersected by a transversal, a pair of angles that lie on the same side of the transversal and on the same sides of each of the other two lines
origin	the point where the *x*-axis and *y*-axis intersect on the coordinate plane; (0, 0)
parallelogram	a quadrilateral with two pairs of parallel sides
quadrant	the *x*- and *y*-axes divide the coordinate plane into four regions. Each region is called a quadrant
segment	a part of a line between two endpoints
trapezoid	a quadrilateral with at least one pair of parallel sides
vertex	on an angle or polygon, the point where two sides intersect; on a polyhedron, the intersection of three or more faces; on a cone or pyramid, the top point
***x*-axis**	the horizontal axis on a coordinate plane
***y*-axis**	the vertical axis on a coordinate plane

center of rotation	the point about which a figure is rotated
congruent	having the same size and shape; the symbol for congruent is ≅
image	a figure resulting from a transformation
line of reflection	the line across which a figure is reflected
preimage	the original figure in a transformation
reflection	a transformation of a figure that flips the figure across a line
rotation	a transformation in which a figure is turned around a point
transformation	a change in the size or position of a figure
translation	a movement (slide) of a figure along a straight line

Lesson 1.1 Investigate Transformations

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations:

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can describe what happens to the sides and angles of a figure when it is transformed.

Learning Objective

Explore and observe the effects of rigid motions on a figure.

Language Objective

Explain how sliding, turning, and flipping a figure affects its shape, size, and direction the shape faces.

Vocabulary

Review: trapezoid
New: transformation

Lesson Materials

ruler, protractor

Lesson 1.2 Explore Translations

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations:

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can translate figures, describe the translations using words and mapping notation, and determine an algebraic rule for translating a figure on a coordinate plane.

Learning Objective

Describe translations and their effects on a figure.

Language Objective

Explain how translations affect figures.

Vocabulary

Review: coordinate plane, segment, vertex
New: image, preimage, translation

Lesson Materials

protractor, ruler; grid paper (Teacher Resource Masters)

Lesson 1.3 Explore Reflections

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations.

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can reflect a figure over either axis in the coordinate plane and describe the reflection algebraically.

Learning Objective

Describe reflections and their effects on a figure.

Language Objective

Understand and explain how to reflect figures on the coordinate plane, and describe the effects of a reflection algebraically.

Vocabulary

Review: corresponding angles, parallelogram, quadrant, *x*-axis, *y*-axis
New: line of reflection, reflection

Lesson Materials

ruler, protractor, grid paper, Coordinate Plane (Teacher Resource Masters)

Lesson 1.4 Explore Rotations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations.

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can identify and perform rotations, and describe a rotation on a coordinate plane algebraically.

Learning Objective

Recognize and perform rotations. Describe rotations algebraically. Understand that rotating a figure produces an image that is congruent to the preimage.

Language Objective

Describe rotations and their effects on a figure.

Vocabulary

Review: origin
New: center of rotation, rotation

Lesson Materials

ruler, protractor; grid paper, Coordinate Plane (Teacher Resource Masters)

Lesson 1.5 Understand and Recognize Congruent Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can determine congruence by performing or describing a sequence of transformations that maps one figure onto another.

Learning Objective

Perform and describe sequences of transformations on figures.

Language Objective

Describe sequences of transformations on figures.

Vocabulary

New: congruent

Lesson Materials

ruler

Unit 1: Transform and Construct Geometric Figures

Unit 1 Project: A Puzzling Transformation
Unit 1 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 2: Draw and Analyze Two-Dimensional Figures

Recommended Pacing with Assessments: 9 Days

Module 2 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and drew lines of symmetry. Students found areas of figures by decomposing those figures into triangles. Students found volumes of right rectangular prisms. Students understood the concept of a ratio. Students used ratio and rate reasoning to solve real-world and mathematical problems. Students used angle relationships to solve problems.	Students draw and construct circles and other figures with given conditions using technology and freehand. Students analyze how many, if any, triangles or quadrilaterals can be made from given side lengths or angle measures. Students demonstrate the rule that the sum of the three angles of a triangle equals 180 degrees. Students solve problems of length and area using scale drawings.	Students will use informal arguments to establish facts about the angle sum and exterior angle of triangles. Students will use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane. Students will compare two different proportional relationships represented in different ways. Students will understand similarity.

Module 2 Vocabulary

corresponding sides	matching sides of two or more polygons
proportional relationship	a relationship between two quantities in which the ratio of one quantity to the other quantity is constant
diameter	a line segment that passes through the center of a circle and has endpoints on the circle; or the length of that segment
radius	a line segment with one endpoint at the center of the circle and the other endpoint on the circle; or the length of that segment
scale	the ratio between two sets of measurements
scale drawing	a drawing that uses a scale to make an object smaller than or larger than the real object

Lesson 2.1 Draw Shapes with Given Coordinates

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

• Use appropriate tools strategically.
• Look for and make use of structure.

I Can Objective

I can inscribe triangles in circles and draw geometric figures meeting given conditions.

Learning Objective

Draw and construct figures using technology and freehand with given conditions.

Language Objective

Describe figures using the words *circumference*, *radius*, *diameter*, *symmetry*, *parallel*, and *perpendicular*.

Vocabulary

New: diameter, radius

Lesson Materials

ruler, compass, protractor

Lesson 2.2 Draw and Construct Triangles Given Side Lengths

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Use appropriate tools strategically.

I Can Objective

I can determine whether three lengths could be side lengths of a triangle, and, given two side lengths, I can find the range of possible lengths for the third side.

Learning Objective

Determine how many triangles or quadrilaterals can be made given the side lengths: none, one, or many.

Language Objective

Understand and be able to explain whether you can form no triangle, one triangle, or many triangles from three given side lengths. Understand and be able to explain whether you can form no quadrilateral, one quadrilateral, or many quadrilaterals from four given side lengths.

Lesson Materials

ruler, compass

Lesson 2.3 Draw and Construct Triangles Given Angle Measures

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can use tools to construct triangles when given angle measures and determine if no triangle or many triangles can be formed.

Learning Objective

Determine how many triangles can be made given the angle measures: none, one, or many.

Language Objective

Describe angle measures and the relationship they form in a triangle.

Lesson Materials

ruler, protractor, compass; grid paper (Teacher Resource Masters)

Lesson 2.4 Draw and Analyze Shapes to Solve Problems

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

I Can Objective

I can draw and analyze shapes, including circles and triangles, to solve real-world problems.

Learning Objective

Draw, construct, and analyze two-dimensional figures to solve real-world problems.

Language Objective

Use the terms *center*, *diameter*, and *radius* in explaining how to construct and analyze circles.

Lesson Materials

ruler, protractor, compass; grid paper (Teacher Resource Masters)

Lesson 2.5 Practice Proportional Reasoning with Scale Drawings

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can use a scale to make a scale drawing of a rectangular object.

Learning Objective

Use scale drawings to solve problems.

Language Objective

Students will explain how to use and interpret scale drawings.

Vocabulary

Review: corresponding angles, proportional relationship
New: scale, scale drawing

Unit 1: Transform and Construct Geometric Figures

Unit 1 Project: A Puzzling Transformation
Unit 1 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 3: Transformations and Similarity

Recommended Pacing with Assessments and Performance Task: 9 Days

Module 3 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students drew polygons in the coordinate plane. Students drew geometric shapes with given conditions. Students solved problems involving scale drawings. Students used coordinates to find the length of a horizontal or vertical segment.	Students learn to describe the effects of dilations. Students dilate images on and off the coordinate plane. Students understand that dilations produce similar figures and the difference between similarity and congruence. Students describe a sequence of transformations that exhibits the similarity between two given figures. Students decide whether two given figures are similar.	Students will verify the properties of dilations. Students will understand dilations and scale factor. Students will prove all circles are similar. Students will use similarity to derive formulas. Students will establish the Angle-Angle Criterion. Students will solve problems and prove relationships in geometric figures.

Module 3 Vocabulary

enlargement	an increase in the size of all dimensions in the same proportion
similar	figures with the same shape but not necessarily the same size
center of dilation	the point of intersection of lines through each pair of corresponding vertices in a dilation
dilation	a transformation that enlarges or reduces a figure
reduction	a decrease in the size of all dimensions in the same proportions
scale factor	the ratio used to enlarge or reduce similar figures

Lesson 3.1 Investigate Reductions and Enlargements

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes
• Use appropriate tools strategically.
• Look for and make use of structure.

I Can Objective
I can identify and perform enlargements and reductions.

Learning Objective
Perform enlargements and reductions. Understand the image that is the result of enlarging or reducing a preimage is not congruent to the preimage.

Language Objective
Enlarge and reduce images on and off the coordinate plane and describe the effects of these transformations on the figure.

Vocabulary
Review: enlargement
New: reduction

Lesson Materials
rulers, protractors; grid paper (Teacher Resource Masters)

Lesson 3.2 Explore Dilations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can identify and perform dilations given a scale factor and center of dilation, perform a dilation on a coordinate plane, and identify an algebraic rule for the dilation.

Learning Objective
Describe and apply the properties of dilations. Understand and find the scale factor and center of dilation, both on and off the coordinate plane.

Language Objective
Dilate images on and off the coordinate plane and recognize that angle measures remain the same while side lengths must be proportional.

Vocabulary
New: center of dilation, dilation, scale factor

Lesson Materials
rulers, protractors; grid paper (Teacher Resource Masters)

Lesson 3.3 Understand and Recognize Similar Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can describe a sequence of transformations that exhibits the similarity between two figures.

Learning Objective

Recognize and draw similar figures using transformations.

Language Objective

Draw and identify similar figures and describe transformations that map one similar figure to another.

Vocabulary

New: similar

Lesson Materials

ruler, protractor

Unit 2: Equations and Inequalities in One Variable

Unit 2 Project: The Rhind Papyrus
Unit 2 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 4: Solve Linear Equations

Recommended Pacing with Assessments: 9 Days

Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students applied the properties of operations to generate equivalent expressions. Students solved real-world problems by writing and solving equations of the form $x + p = q$ and $px = q$, where p, q, and x are all non-negative rational numbers.	Students write and solve multi-step one-variable linear equations. Students interpret solutions of linear equations in context. Students explore and interpret equations. Students solve real-world problems involving multi-step linear equations in one variable. Students use supplementary, complementary, vertical, and adjacent angles in multi-step problems.	Students will write equations in one variable and use them to solve problems. Students will solve linear equations with coefficients represented by letters. Students will know and apply the properties of integer exponents. Students will use square and cube root symbols to solve equations. Students will evaluate square and cube roots.

Module 4 Vocabulary

coefficient	the number that is multiplied by the variable in an algebraic expression
common denominator	a denominator that is the same in two or more fractions
Distributive Property	for all real numbers a, b, and c, $a(b + c) = ab + ac$, and $a(b - c) = ab - ac$
expression	a mathematical phrase that contains operations, numbers, and/or variables
isolate the variable	to get a variable alone on one side of an equation or inequality in order to solve the equation or inequality
like terms	terms that have the same variable(s) raised to the same exponent
multiple	the product of any number and any nonzero whole number is a multiple of that number
solution of an equation	a value or values that make an equation true
substitute	to replace a variable with a number or another expression in an algebraic expression

adjacent angles	angles in the same plane that have a common vertex and a common side
complementary angles	two angles whose measures have a sum of 90°
infinitely many solutions	occurs when every value of *x* creates a true mathematical statement
no solution	occurs when every value of *x* creates a false mathematical statement
supplementary angles	two angles whose measures have a sum of 180°
vertical angles	a pair of opposite congruent angles formed by intersecting lines

Lesson 4.1 Write Two-Step Equations for Situations

Build Conceptual Understanding – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can write two-step equations for various situations.

Learning Objective

Represent a real-world situation with an equation.

Language Objective

Write equations of the form $px + q = r$ and $p(x + q) = r$ to represent real-world situations.

Vocabulary

Review: expression, like terms

Lesson Materials

algebra tiles, equation mat

Lesson 4.2 Apply Two-Step Equations to Solve Real-World Problems

Connect Concepts and Skills – 1 Day

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can apply two-step equations to solve a variety of problems.

Learning Objective

Solve real-world problems using an equation.

Language Objective

Explain how to solve real-world situations that can be modeled with equations.

Vocabulary

Review: solution of an equation

Lesson Materials

algebra tiles, equation mat; number line (Teacher Resource Masters)

Lesson 4.3 Solve Multi-Step Linear Equations

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can solve linear equations with integer and rational number coefficients.

Learning Objective

Use algebraic properties to solve one-variable linear equations.

Language Objective

Explain how to solve one-variable linear equations.

Vocabulary

Review: coefficient, common denominator, Distributive Property, isolate the variable, multiple

Lesson Materials

algebra tiles, equation mat

Lesson 4.4 Examine Special Cases

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can recognize and solve linear equations that have no solution, one solution, or infinitely many solutions.

Learning Objective

Recognize and interpret linear equations that have no solution or infinitely many solutions.

Language Objective

Explain how to recognize linear equations that have no solution or infinitely many solutions.

Vocabulary

Review: substitution

New: infinitely many solutions, no solution

Lesson Materials

algebra tiles, equation mat

Lesson 4.5 Apply Linear Equations

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Solve linear equations in one variable.

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can solve equations and interpret solutions in context.

Learning Objective

Solve and apply linear equations in one variable.

Language Objective

Explain how to solve and apply linear equations in one variable.

Vocabulary

New: adjacent angles, complementary angles, supplementary angles, vertical angles

Unit 2: Equations and Inequalities in One Variable

Unit 2 Project: The Rhind Papyrus
Unit 2 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 5: Solve Problems Using Inequalities

Recommended Pacing with Assessments and Performance Task: 8 Days

Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote inequalities of the form $x > c$ or $x < c$ to represent constraints or conditions in a real-world or mathematical problem. Students solved equations as a process of answering a question. Students used variables to represent quantities in a real-world or mathematical problem, and constructed simple equations to solve problems by reasoning about the quantities.	Students solve one- and two-step inequalities for real-world problems. Students write and solve inequalities in the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Students graph the solution sets of inequalities and interpret them in the context of the problem.	Students will solve compound inequalities. Students will graph and solve inequalities in two variables.

Module 5 Vocabulary

inequality	a mathematical sentence that shows the relationship between quantities that are not equivalent
number line	a line used to plot real numbers, which include rational numbers and irrational numbers
solution of an inequality	a value or values that make an inequality true
rate of change	a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable

Lesson 5.1 Understand and Apply Properties to Solve One-Step Inequalities

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can write and solve one-step inequalities.

Learning Objective

Apply properties to solve one-step inequalities.

Language Objective

Explain how to set up and solve one-step inequalities.

Vocabulary

Review: inequality, number line, solution of an inequality
New: rate of change

Lesson Materials

algebra tiles, equation mat; number lines (Teacher Resource Masters)

Lesson 5.2 Write Two-Step Inequalities for Situations

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can write two-step inequalities to solve problems.

Learning Objective

Write two-step inequalities to represent situations.

Language Objective

Use the terms *greater than*, *less than*, *greater than or equal to*, and *less than or equal to* in the context of inequalities to represent situations.

Lesson Materials

algebra tiles, equation mat; number lines (Teacher Resource Masters)

Lesson 5.3 Apply Two-Step Inequalities to Solve Problems

Apply and Practice – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can write and solve two-step inequalities to solve problems.

Learning Objective

Write, solve, and graph one-step and two-step inequalities to solve problems in context.

Language Objective

Write inequalities to represent situations and interpret the solutions in context.

Lesson Materials

algebra tiles, equation mat

Unit 3: Similarity, Slope, and Linear Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 6: Angle Relationships

Recommended Pacing with Assessments: 8 Days

Module 6 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and used supplementary, complementary, vertical, and adjacent angles in multi-step problems. Students understood and described similar figures.	Students establish facts about the angle sum and exterior angle of triangles. Students use angle measures to determine whether two triangles are similar. Students show that corresponding angles, alternate exterior angles, and alternate interior angles are congruent, and that same-side interior or exterior angles are supplementary. Students use these facts to find missing angle measures.	Students will prove theorems about triangles. Students will use triangles to solve problems. Students will work with similar figures. Students will prove theorems about lines and angles. Students will solve problems using theorems about lines and angles.

Module 6 Vocabulary

alternate exterior angle	for two lines intersected by a transversal, a pair of angles that lie on opposite sides of the transversal and outside the other two lines
alternate interior angles	for two lines intersected by a transversal, a pair of nonadjacent angles that lie on opposite sides of the transversal and between the other two lines
Angle-Angle Similarity Postulate	the postulate that states that two triangles are similar if they have two sets of corresponding angles that are congruent
corresponding angles	for two lines intersected by a transversal, a pair of angles that lie on the same side of the transversal and on the same sides of the other two lines
exterior angle	an angle formed by one side of a polygon and the extension of an adjacent side
Exterior Angle Theorem	the theorem that states that the measure of an exterior angle of a triangle is greater than either of the measures of the remote interior angles
remote interior angle	an interior angle of a polygon that is not adjacent to the exterior angle
same-side exterior angles	a pair of angles on the same side of a transversal but outside the parallel lines

same-side interior angles	a pair of angles on the same side of a transversal and between two lines intersected by the transversal
transversal	a line that intersects two or more lines
Triangle Sum Theorem	the theorem that states that the measures of the angles in a triangle add up to 180°

Lesson 6.1 Develop Angle Relationships for Triangles

Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can find an unknown angle measure in a triangle.

Learning Objective

Use angle relationships in triangles.

Language Objective

Use and describe angle relationships in triangles.

Vocabulary

New: exterior angle, Exterior Angle Theorem, remote interior angle, Triangle Sum Theorem

Lesson Materials

ruler, protractor; Polygons: Triangles (Teacher Resource Masters)

Lesson 6.2 Investigate Angle-Angle Similarity

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Construct viable arguments and critique the reasoning of others.
• Model with mathematics.

I Can Objective

I can use angle-angle similarity to test triangles for similarity and find unknown angle measures.

Learning Objective

Identify whether two triangles are similar, given angle measures in the triangles. Find unknown angle measures in triangles known to be similar.

Language Objective

Explain how to identify two similar triangles based on the presence of two pairs of congruent angles.

Vocabulary

New: Angle-Angle Similarity Postulate

Lesson Materials

ruler, protractor

Lesson 6.3 Explore Parallel Lines Cut by a Transversal

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.
• Look for and make use of structure.

I Can Objective

I can identify the relationship between angle pairs as either supplementary or congruent.

Learning Objective

Find unknown angle measures when parallel lines are cut by a transversal.

Language Objective

Identify the angle pairs made when parallel lines are intersected by a transversal and say whether the angles are congruent or supplementary.

Vocabulary

New: alternate exterior angles, alternate interior angles, corresponding angles, same-side exterior angles, same-side interior angle, transversal

Lesson Materials

ruler, protractor

Unit 3: Similarity, Slope, and Linear Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 7: Proportional Relationships

Recommended Pacing with Assessments: 7 Days

Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved problems involving scale drawings. Students recognized and represented proportional relationships. Students understood the concept of a unit rate. Students analyzed relationships using graphs.	Students use similar triangles to understand slope. Students derive the equation $y = mx$. Students write an equation of a proportional relationship. Students graph proportional relationships. Students identify and use unit rates. Students compare proportional relationships presented in different representations.	Students will create equations to represent relationships between quantities. Students will understand the concept of a function. Students will understand the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Students will interpret statements that use function notation in terms of a context. Students will compare properties of two functions.

Module 7 Vocabulary

unit rate	a rate in which the second quantity in the comparison is one unit
continuous graph	a graph made up of connected lines or curves
discrete graph	a graph made up of unconnected points
hypotenuse	in a right triangle, the side opposite the right angle
legs	in a right triangle, the sides that include the right angle; in an isosceles triangle, the pair of congruent sides
linear equation	an equation whose solutions form a straight line on a coordinate plane
rise	the vertical change when the slope of a line is expressed as the rise-over-run ratio
run	the horizontal change when the slope of a line is expressed as the rise-over-run ratio
slope	a measure of the steepness of a line on a graph: the rise divided by the run
$y = mx$	equation for a line that passes through the origin

Lesson 7.1 Explain Slope with Similar Triangles

Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can determine the slope of a line and use it to find additional points on the line.

Learning Objective

Relate right triangles to the coordinates of a line going through the origin, and compare persistent features of the triangles to persistent features of the line.

Language Objective

Describe how to use similar right triangles to verify that the slope of a line is constant.

Vocabulary

Review: unit rate
New: hypotenuse, legs, rise, run, slope

Lesson Materials

protractor, ruler

Lesson 7.2 Derive $y = mx$

Connect Concepts and Skills – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can write the equation of a line given a graph or a table of values.

Learning Objective

Write the equation of a proportional relationship.

Language Objective

Explain how to identify the unit rate of change and write an equation that represents a proportional relationship.

Vocabulary

New: linear equation, $y = mx$

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 7.3 Graph, Interpret, and Compare Proportional Relationships

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Mathematical Practices and Processes

- Model with mathematics.
- Reason abstractly and quantitatively.

I Can Objective

I can graph proportional relationships and interpret the unit rate as the slope of the graph.

Learning Objective

Graph proportional relationships. Interpret unit rate as the slope of the graph of a proportional relationship.

Language Objective

Explain how to find the unit rate of a proportional relationship from graphs and tables and compare proportional relationships given in different forms.

Vocabulary

New: continuous graph, discrete graph

Lesson Materials

ruler, Grid of Quadrant 1 (Teacher Resource Masters)

Unit 3: Similarity, Slope, and Linear Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 8: Understand and Analyze Functions

Recommended Pacing with Assessments: 12 Days

Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students derived $y = mx$. Students recognized, represented, and compared proportional relationships. Students analyzed the relationship between dependent and independent variables. Students interpreted unit rate as slope. Students solved problems involving unit pricing and speed. Students understood and graphed functions.	Students learn to identify functions and give examples of functions that are not linear. Students use functions to solve real-world problems. Student derive and interpret $y = mx + b$. Students interpret the meaning of the slope and *y*-intercept in a context. Students construct a function. Students compare functions. Students sketch, analyze, and describe a graph that exhibits the qualitative features of a function.	Students will understand functions and use function notation. Students will relate the domain of a function to its graph. Students will graph linear functions and show intercepts. Students will write a function in different forms to reveal and explain different properties. Students will compare properties of two functions. Students will interpret key features of functions. Students will recognize that sequences are functions.

Module 8 Vocabulary

domain	the set of all possible input values of a function
function	an input-output relationship that has exactly one output for each input
input	the value substituted into an expression or function
linear function	a function whose graph is a straight line
nonlinear function	a function whose graph is not a straight line
output	the value that results from the substitution of a given input into an expression or function
range	the set of all possible output values of a function

relation	a set of ordered pairs
slope-intercept form	a linear equation written in the form $y = mx + b$, where m represents slope and b represents the y-intercept
vertical line test	a test used to determine whether a relation is a function. If any vertical line crosses the graph of a relation more than once, the relation is not a function
***y*-intercept**	the y-coordinate of the point where the graph of a line crosses the y-axis

Lesson 8.1 Understand and Graph Functions

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can form coordinate-pair tables from real-life scenarios, graph coordinate pairs from a table, and identify functions from both tables and graphs.

Learning Objective

Display a relationship between two variables.

Language Objective

Graph functions, determine domain and range, and be able to differentiate functions from non-functions given data or a graph.

Vocabulary

New: domain, function, input, output, range, relation, vertical line test

Lesson 8.2 Derive $y = mx + b$

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can derive the equation for a line in the form $y = mx + b$ given the slope of the line and a point.

Learning Objective

Write the equation of a linear function.

Language Objective

Explore lines to derive the equation $y = mx + b$, and be able to use the equation in all four quadrants to describe points on lines.

Vocabulary

New: linear function, nonlinear function, slope-intercept form, y-intercept

Lesson 8.3 Interpret Rate of Change and Initial Value

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Practices and Processes

- Model with mathematics.
- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can find and interpret initial value and rate of change of a function.

Learning Objective

Interpret the slope and y-intercept of a line.

Language Objective

Explain the connection between slope, rate of change, and m; and the connection between y-intercept, initial value, and b.

Lesson Materials

Grid of Quadrant 1 (Teacher Resource Masters)

Lesson 8.4 Construct and Compare Functions

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can construct and compare functions represented in equations, tables, graphs, or verbal descriptions.

Learning Objective

Construct a function to model, understand, and analyze a linear relationship.

Language Objective

Explain how to determine if a function is discrete or continuous.

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 8.5 Describe and Sketch Nonlinear Functions

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can convert between a verbal description of a function and its graph, and between a graph and a verbal description of a function.

Learning Objective

Sketch and analyze a graph that exhibits the qualitative features of a function.

Language Objective

Sketch graphs given the description of the graph. Describe a function given its graph.

Unit 3: Similarity, Slope, and Linear Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 9: Systems of Linear Equations

Recommended Pacing with Assessments and Performance Task: 12 Days

Module 9 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students graphed points in all four quadrants. Students understood solving an equation as a process of answering a question. Students wrote and solved one-variable equations. Students used variables to represent quantities in a real-world problem.	Students use graphing to determine the approximate solution to a system. Students solve systems of linear equations by graphing, substitution, and elimination. Students learn to recognize and interpret graphic and algebraic results of solving a system with no solution or infinitely many solutions. Students use systems of two linear equations to represent real-world problems.	Students will solve systems of linear equations exactly and approximately. Students will prove solving by elimination produces a system with the same solutions. Students will represent constraints by systems of equations. Students will interpret solutions as viable or non-viable in a modeling context.

Module 9 Vocabulary

equivalent expressions	expressions that have the same value for all values of the variable
substitute	to replace a variable with a number or another expression in an algebraic expression
elimination	algebraic process of eliminating a variable in a system of equations by combining the equations through addition
solution of a system of equations	a set of values that make all equations in a system true
system of equations	a set of two or more equations that contain two or more variables

Lesson 9.1 Represent Systems by Graphing

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Look for and make use of structure.

I Can Objective

I can graph a pair of linear equations and draw a conclusion from the graph.

Learning Objective

Interpret the graphical representation of two linear equations.

Language Objective

Describe the relationships in a graphical representation of two linear equations.

Lesson Materials

Grid of Quadrant 1, Coordinate Plane (Teacher Resource Masters)

Lesson 9.2 Solve Systems by Graphing

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Attend to precision.

I Can Objective

I can solve a system of linear equations by graphing.

Learning Objective

Solve a system of two linear equations by graphing.

Language Objective

Describe how to use graphing to solve a system of two linear equations.

Vocabulary

New: solution of a system of equations, system of equations

Lesson Materials

ruler; grid paper, Coordinate Plane (Teacher Resource Masters)

Lesson 9.3 Solve Systems by Substitution

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can solve systems of equations by substitution.

Learning Objective

Use substitution to solve a system of two linear equations.

Language Objective

Explain how to solve a system of two linear equations using substitution.

Vocabulary

Review: substitute

Lesson Materials

algebra tiles, equation mat

Lesson 9.4 Solve Systems by Elimination

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can solve a system of linear equations by elimination.

Learning Objective

Use elimination to solve a system of two linear equations.

Language Objective

Explain how a variable is eliminated by multiplying equations by a factor and then adding equations to solve a system of equations.

Vocabulary

Review: equivalent expressions
New: elimination

Lesson Materials

Coordinate Plane (Teacher Resource Masters)

Lesson 9.5 Examine Special Systems

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can identify the number of solutions to a system of linear equations in any form.

Learning Objective

Recognize and interpret systems of two linear equations that have no solution or infinitely many solutions.

Language Objective

Students relate the solutions of a system of equations to the intersections of lines on the coordinate plane.

Lesson Materials

Coordinate Plane (Teacher Resource Masters)

Lesson 9.6 Apply Systems of Equations

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can write and solve a system of equations to solve a real-world problem.

Learning Objective

Use systems of two linear equations to solve real-world problems.

Language Objective

Students will be able to read and interpret a description of a context and translate this into a system of linear equations to solve real-world problems.

Lesson Materials

algebra tiles; grid paper, Word Description graphic organizer (Teacher Resource Masters)

Unit 4: Data Analysis and Sampling

Unit 4 Project: A Birthday Puzzle
Unit 4 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 10: Scatter Plots

Recommended Pacing with Assessments: 7 Days

Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students displayed and summarized numerical data in plots on a number line in context. Students understood that statistics can be used to gain information about a population. Students informally assessed the degree of visual overlap of two numerical data distributions. Students summarized numerical data sets in relation to their context.	Students recognize that lines can be used to model the pattern shown in a scatter plot. Students sketch a linear model for a data set displayed in a scatter plot. Students understand the influence of outliers on the trend line of a data set. Students compare linear models to determine which is the best fit for given data. Students interpret rate of change and initial value of a linear function.	Students will fit functions to data displayed in a scatter plot. Students will distinguish between correlation and causation and evaluate reports. Students will compute and interpret the correlation coefficient and interpret slope in context.

Module 10 Vocabulary

data set	A set of information collected about people or things, often to draw conclusions about them
outlier	a value much greater or much less than the others in a data set
association	the description of the relationship between two data sets
cluster	set of closely grouped data
linear association	when data points roughly lie along a line
negative association	two data sets have a negative association if one set of data values increases while the other decreases
no association	two data sets have no association when there is no relationship between their data values
nonlinear association	an association between two variables in which the data do not have a linear trend
positive association	two data sets have a positive association when their data values increase or decrease together

scatter plot	a graph with points plotted to show a possible relationship between two sets of data
trend line	a line on a scatter plot that helps show the association between data sets more clearly

Lesson 10.1 Construct Scatter Plots and Examine Association

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.

I Can Objective

I can construct a scatter plot, determining whether an association is positive or negative, strong or weak, and nonlinear or linear. I can recognize outliers and clusters.

Learning Objective

Display and analyze data with two variables.

Language Objective

Use the adjectives *positive*, *negative*, *strong*, and *weak* to describe the association between two variables shown in a scatter plot.

Vocabulary

Review: data set, outlier
New: association, cluster, linear association, negative association, no association, nonlinear association, positive association, scatter plot

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 10.2 Draw and Analyze Trend Lines

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can draw a trend line and informally assess a trend line by judging the closeness of the data points to the line.

Learning Objective

Use trend lines to describe a linear relationship between two variables.

Language Objective

Explain how to use a line to model the pattern shown by a data set.

Vocabulary

New: trend line

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 10.3 Interpret Linear Data in Context

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can determine the equation of a line to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Learning Objective

Use scatter plots and trend lines to interpret linear data in context.

Language Objective

Explain how to use a scatter plot and a trend line to determine reasonable values for data in context.

Lesson Materials

grid paper (Teacher Resource Masters)

Unit 4: Data Analysis and Sampling

Unit 4 Project: A Birthday Puzzle
Unit 4 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 11: Proportional Reasoning with Samples

Recommended Pacing with Assessments: 6 Days

Module 11 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students explored statistical data collection. Students learned how to choose a representative sample. Students analyzed and graphed statistical data. Students gave quantitative measures of center and variability.	Students understand how to select a representative sample. Students understand the ways that a sample could be biased. Students understand that proportional reasoning can be used to make inferences about a population from a random sample. Students use proportional reasoning to make inferences about a population. Students use data from a random sample to draw inferences about a population.	Students will compare data sets by measures of center. Students will compare data sets by measures of variability. Students will construct and interpret data in tables, including two-way frequency tables and two-way relative frequency tables. Students will use relative frequencies calculated for rows or columns to describe possible association between two variables.

Module 11 Vocabulary

bias	when a sample does not accurately represent the population
population	the entire group of objects or individuals considered for a survey
random sample	a sample in which each individual or object in the entire population has an equal chance of being selected
representative sample	a sample that has the same characteristics of the population
sample	a part of the population

Lesson 11.1 Understand Representative Samples

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can identify the population and sample for a given survey scenario and say whether a sample is random. I can determine whether a sample is likely to be representative of the population.

Learning Objective

Understand populations, random samples, and how to select a representative sample.

Language Objective

Properly use the terms *population* and *sample* when describing data collection.

Vocabulary

New: bias, population, random sample, representative sample, sample

Lesson Materials

Fraction Circles (Teacher Resource Masters)

Lesson 11.2 Make Inferences from a Random Sample

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Mathematical Practices and Processes

• Attend to precision.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can use proportional reasoning to make inferences about populations based on the results of a random sample.

Learning Objective

Use a random sample to make inferences about a population.

Language Objective

Properly use the terms *population* and *sample* when making inferences.

Lesson Materials

Fraction Circles (Halves Through Sixths),
Grid of Quadrant I (Teacher Resource Masters)

Lesson 11.3 Make Inferences from Repeated Random Samples

Apply and Practice – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Attend to precision.

I Can Objective

I can use multiple random samples of the same size from a population to make inferences about a survey result.

Learning Objective

Understand that repeatedly sampling a population with the same size random sample will cause the data to vary.

Language Objective

Properly use the terms *population*, *sample*, and *representative sample* when discussing data from repeated samples.

Lesson Materials

number lines (Teacher Resource Masters)

Unit 4: Data Analysis and Sampling

Unit 4 Project: A Birthday Puzzle
Unit 4 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 12: Use Statistics and Graphs to Compare Data

Recommended Pacing with Assessments: 6 Days

Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students displayed data in dot plots and box plots. Students calculated five key values: minimum, lower quartile, median, upper quartile, and maximum. Students summarized and analyzed data using measures of center and measures of variability.	Students compare the shapes, centers, and spreads of data displayed in dot plots and box plots. Students draw conclusions about populations based on sample data displayed in dot plots and box plots. Students use measures of center and variability, including mean absolute deviation for data from random samples to draw informal comparisons about the populations.	Students will construct two-way frequency tables and relative frequency tables. Students will use relative frequencies to describe an association between two variables.

Module 12 Vocabulary

box plot	a graph that shows how data are distributed by using the median, quartiles, least value, and greatest value
interquartile range	the difference between the upper and lower quartiles in a box plot
lower quartile	the median of the lower half of a set of data
mean	the sum of the items in a set of data divided by the number of items in the set; also called *average*
mean absolute deviation	the mean distance between each data value and the mean of the data set
median	the middle number, or the mean (average) of the two middle numbers, in an ordered set of data
range	the difference between the greatest and least values in a data set
upper quartile	the median of the upper half of a set of data

Lesson 12.1 Compare Center and Spread of Data Displayed in Dot Plots

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can compare two data sets displayed in dot plots and make inferences about two populations.

Learning Objective

Compare the center and spread of data displayed in dot plots.

Language Objective

Use the terms *center* and *spread* to describe key features of data sets displayed in dot plots.

Vocabulary

Review: mean, median, range

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 12.2 Compare Center and Spread of Data Displayed in Box Plots

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can compare two data sets displayed in box plots and make inferences about two populations.

Learning Objective

Compare data displayed in box plots, and use these comparisons to draw inferences about two populations.

Language Objective

Use the terms *box plot*, *range*, and *interquartile range* when comparing key features of data sets.

Vocabulary

Review: box plot, interquartile range, lower quartile, upper quartile

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 12.3 Compare Means Using Mean Absolute Deviation and Repeated Sampling

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can use the means and MADS to assess the amount of visual overlap of two numerical data distributions.

Learning Objective

Use means and MADs to compare two populations.

Language Objective

Use the terms *mean absolute deviation* and *mean* to compare differences in center and spread of two populations.

Vocabulary

Review: mean absolute deviation

Lesson Materials

Number lines (Teacher Resource Masters)

Unit 4: Data Analysis and Sampling

Unit 4 Project: A Birthday Puzzle
Unit 4 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 13: Two-Way Tables

Recommended Pacing with Assessments and Performance Task: 7 Days

Module 13 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students used data from a random sample to draw inferences about a population. Students collected and displayed numerical data and interpreted data in context. Students constructed and interpreted frequency tables.	Students construct and interpret two-way frequency tables. Students determine whether there is an association between events. Students construct and interpret two-way relative frequency tables. Students calculate and interpret conditional relative frequencies.	Students will understand characteristics of independent events. Students will understand, recognize, and explain the concept of conditional probability. Students will determine if two events are independent.

Module 13 Vocabulary

conditional relative frequency	the ratio of a joint relative frequency to a related marginal relative frequency in a two-way table
event	an outcome or set of outcomes of an experiment or situation
frequency	the number of times the value appears in the data set
joint relative frequency	the frequency in a particular category divided by the total number of data values
marginal relative frequency	the sum of the joint relative frequencies in a row or column of a two-way table
relative frequency	the frequency of a specific data value divided by the total number of data values in the set
two-way relative frequency table	a two-way table that displays relative frequencies
two-way table	a table that displays two-variable data by organizing it into rows and columns

Lesson 13.1 Construct and Interpret Two-Way Frequency Tables

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can construct a two-way table summarizing data, complete a table given partial data, and interpret data to determine whether there is an association between two variables.

Learning Objective

Interpret data by constructing two-way frequency tables.

Language Objective

Describe and justify conclusions derived from constructing and interpreting two-way frequency tables.

Vocabulary

New: two-way table

Lesson 13.2 Analyze and Interpret Two-Way Relative Frequency Tables

Apply and Practice – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can construct two-way frequency tables, find relative frequencies, and use them to determine if there is an association between two variables.

Learning Objective

Analyze and interpret two-way relative frequency tables.

Language Objective

Explain how to construct and analyze two-way relative frequency tables.

Vocabulary

New: conditional relative frequency, event, frequency, joint relative frequency, marginal relative frequency, relative frequency, two-way relative frequency table

Unit 5: Applications of Real Numbers and Exponents

Unit 5 Project: The Wheel of Theodorus
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 14: Real Numbers

Recommended Pacing with Assessments: 7 Days

Module 14 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood a rational number as a point on the number line. Students converted a rational number to a decimal. Students classified fractions and decimals as rational numbers. Students graphed rational numbers on a number line. Students wrote and evaluated numerical expressions involving whole-number exponents.	Students determine whether a number is rational or irrational. Students write rational numbers as decimals or fractions. Students evaluate square roots and cube roots and solve equations. Students identify decimal estimates of square roots and cube roots. Students compare numerical expressions involving roots. Students order a list of real numbers including rational and irrational numbers.	Students will generalize results from operations with rational and irrational numbers. Students will solve simple rational and radical equations in one variable. Students will understand notation for radicals in terms of rational exponents.

Module 14 Vocabulary

cube	a number raised to the third power
rational number	any number that can be expressed as a ratio of two integers
repeating decimal	a decimal in which one or more digits repeat infinitely
terminating decimal	a decimal number that ends, or terminates
cube root	the cube root of a positive number p is x when x^3 = p
irrational number	a number that cannot be expressed as a ratio of two integers or as a repeating or terminating decimal
perfect cube	a cube of a whole number
perfect square	a square of a whole number
principal square root	the nonnegative square root of a number

radical symbol	the symbol used to represent the nonnegative square root of a number
real number	a rational or irrational number
square root	the square root of a positive number *p* is *x* when $x^2 = p$

Lesson 14.1 Understand Rational and Irrational Numbers

Connect Concepts and Skills – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Use appropriate tools strategically.

I Can Objective

I can determine whether a number is rational and write a given rational number as a fraction.

Learning Objective

Determine if a number is rational.

Language Objective

Use the terms *rational*, *irrational*, *terminating decimal*, and *repeating decimal* to describe numbers.

Vocabulary

Review: rational number, repeating decimal, terminating decimal

New: irrational number

Lesson 14.2 Investigate Roots

Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can evaluate square roots and cube roots.

Learning Objective

Evaluate square and cube roots.

Language Objective

Describe how to evaluate square and cube roots.

Vocabulary

Review: cube

New: cube root, perfect cube, perfect square, principal square root, radical symbol, square root

Lesson Materials

ruler, protractor

Lesson 14.3 Order Real Numbers

Apply and Practice – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Mathematical Practices and Processes

- Attend to precision.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can accurately order a list of real numbers containing fractions, decimals, and irrational numbers.

Learning Objective

Order a list of real numbers consisting of both rational and irrational numbers.

Language Objective

Use the terms *less than* or *greater than* to compare two real numbers.

Vocabulary

New: real numbers

Unit 5: Applications of Real Numbers and Exponents

Unit 5 Project: The Wheel of Theodorus
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 15: The Pythagorean Theorem

Recommended Pacing with Assessments: 8 Days

Module 15 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood irrational numbers and roots. Students drew geometric shapes with given conditions.	Students prove the Pythagorean Theorem and its converse. Students solve basic problems using the Pythagorean Theorem. Students apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Students apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.	Students will prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$. Students will derive the equation of a circle using the Pythagorean Theorem. Students will prove theorems about triangles. Students will formally prove the Pythagorean Theorem using triangle similarity. Students will use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Module 15 Vocabulary

cone	a three-dimensional figure with one vertex and one circular base
Pythagorean Theorem	In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
Pythagorean triple	a set of three positive integers a, b, and c such that $a^2 + b^2 = c^2$

Lesson 15.1 Prove the Pythagorean Theorem and Its Converse

Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Explain a proof of the Pythagorean Theorem and its converse.

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can prove the Pythagorean Theorem and its converse, use the Pythagorean Theorem to find missing side lengths of right triangles, and identify a Pythagorean triple.

Learning Objective

Prove and apply the Pythagorean Theorem and its converse.

Language Objective

Use the terms *hypotenuse* and *legs* to describe parts of a right triangle. Explain the meaning of a Pythagorean triple and identify Pythagorean triples.

Use the terms converse, hypotenuse, and Pythagorean Theorem to prove that if $a^2 + b^2 = c^2$ is true for a triangle, then the triangle is a right triangle.

Vocabulary

New: Pythagorean Theorem, Pythagorean triple

Lesson Materials

ruler; grid paper, dot paper (Teacher Resource Masters)

Lesson 15.2 Apply the Pythagorean Theorem

Apply and Practice – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can apply the Pythagorean Theorem to solve real-life problems involving the legs and hypotenuse of a right triangle, including problems in three dimensions.

Learning Objective

Use the Pythagorean Theorem to solve real-world problems involving right triangles.

Language Objective

Classify the three sides of a right triangle as legs and the hypotenuse.

Vocabulary

Review: cone

Lesson 15.3 Apply the Pythagorean Theorem in the Coordinate Plane

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective

I can apply the Pythagorean Theorem to find the lengths of line segments on the coordinate plane, including line segments that are part of a composite figure.

Learning Objective

Use the Pythagorean Theorem to determine distance between any two points in the coordinate plane.

Language Objective

Explain how to use the Pythagorean Theorem to find the distance between two points in a coordinate plane.

Lesson Materials

grid paper (Teacher Resource Masters); straightedge

Unit 5: Applications of Real Numbers and Exponents

Unit 5 Project: The Wheel of Theodorus
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 16: Exponents and Scientific Notation

Recommended Pacing with Assessments and Performance Task: 8 Days

Module 16 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote and evaluated numerical expressions involving whole-number exponents. Students applied the properties of operations to generate equivalent expressions. Students solved problems posed with positive and negative rational numbers in any form.	Students develop the properties of integer exponents. Students use the properties of integer exponents to simplify expressions and solve problems. Students translate between standard notation and scientific notation or vice versa. Students compare and compute numbers using scientific notation.	Students will choose and interpret units consistently in formulas. Students will explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents. Students will rewrite expressions involving radicals and rational exponents.

Module 16 Vocabulary

base	the number that is used as a factor when a number is raised to a power
exponent	the number that indicates how many times the base is used as a factor
power	a number produced by raising a base to an exponent
properties of exponents	properties for operations with exponents
scientific notation	a method of writing very large or very small numbers by using powers of 10
standard form of a number	way of writing a number by using digits

Lesson 16.1 Know and Apply Properties of Exponents

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Mathematical Practices and Processes
- Attend to precision.
- Look for and make use of structure.

I Can Objective
I can use properties of integer exponents to simplify expressions.

Learning Objective
Develop and use the properties of integer exponents.

Language Objective
Explain how to develop and use the properties of integer exponents to simplify expressions and solve problems.

Vocabulary
Review: base, exponent, power
New: properties of exponents

Lesson Materials
tape measure

Lesson 16.2 Understand Scientific Notation

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Mathematical Practices and Processes
- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective
I can use scientific notation to describe very large or very small quantities and to compare quantities.

Learning Objective
Express numbers using scientific notation.

Language Objective
Explain how to express numbers using scientific notation.

Vocabulary
New: scientific notation, standard form of a number

Lesson 16.3 Compute with Scientific Notation

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can compute with numbers in scientific notation and choose appropriate units for very large or small quantities.

Learning Objective

Compute with numbers written in scientific notation.

Language Objective

Explain how to compute with numbers written in scientific notation.

Unit 6: Area and Volume

Unit 6 Project: Buffon's Needle
Unit 6 Learning Mindset Focus: Perseverance: Learns Effectively

Module 17: Analyze Figures to Find Circumference and Area

Recommended Pacing with Assessments: 5 Days

Module 17 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students calculated circumference and area of circles. Students found surface area and volume of solids. Students found areas of triangles and special quadrilaterals. Students found the diameter and radius of circles.	Students know and use the formulas for circumference and area of a circle to solve problems. Students solve multi-step real-life and mathematical problems involving positive rational numbers. Students calculate the area of a composite figure.	Students will derive the equation of a circle using the Pythagorean Theorem. Students will solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Students will calculate arc length, sector area, and analyze inscribed and circumscribed circles.

Module 17 Vocabulary

composite figure	a figure made up of simple geometric shapes
circumference	the distance around a circle for one rotation
pi (π)	the ratio of the circumference of a circle to the length of its diameter; $\pi \approx 3.14$ or $\frac{22}{7}$

Lesson 17.1 Derive and Apply Formulas for Circumference

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can find the circumference of a circle when I know either the radius or diameter.

Learning Objective

Derive and apply formulas for circumference.

Language Objective

Explain how to derive and apply formulas for circumference.

Vocabulary

New: circumference, pi (π)

Lesson Materials

compass, ruler

Lesson 17.2 Derive and Apply a Formula for the Area of a Circle

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can use formulas for the area and circumference of a circle to solve problems and informally derive the relationship between the circumference and the area.

Learning Objective

Derive and apply formulas for the area of a circle.

Language Objective

Explain the meaning of the terms *radius*, *diameter*, *circumference*, and *area* of circles.

Lesson Materials

compass; grid paper (Teacher Resource Masters)

Lesson 17.3 Areas of Composite Figures

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Look for and make use of structure.
- Use appropriate tools strategically.

I Can Objective

I can break a composite figure into simple shapes and use area formulas to find its area.

Learning Objective

Use known formulas to calculate the areas of composite figures.

Language Objective

Explain how to calculate the area of a composite figure by breaking it into its component figures and adding their areas.

Vocabulary

Review: composite figure

Unit 6: Area and Volume

Unit 6 Project: Buffon's Needle
Unit 6 Learning Mindset Focus: Perseverance: Learns Effectively

Module 18: Cross Sections, Surface Area, and Volume

Recommended Pacing with Assessments and Performance Task: 11 Days

Module 18 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students described two-dimensional figures produced by slicing a three-dimensional circular solid. Students used the nets to find the surface area of three-dimensional figures. Students found the volume of prisms. Students solved real-world and mathematical problems involving area of two-dimensional composite figures.	Students describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Students solve problems involving area, volume, and surface area. Students find volumes of spheres, cones, and cylinders and use them to solve problems.	Students will derive the equation of a circle using the Pythagorean Theorem. Students will give an informal argument using Cavalieri's principle for the formula for the volume of a sphere. Students will apply concepts of density based on area and volume in modeling situations.

Module 17 Vocabulary

pyramid	a polyhedron with a polygon base and triangular sides that all meet at a common vertex
rectangular prism	a polyhedron whose bases are rectangles and whose other faces are parallelograms
surface area	the sum of the areas of the faces, or surfaces, of a three-dimensional figure
cross section	the intersection of a three-dimensional figure and a plane
cylinder	a three-dimensional figure with two parallel, congruent circular bases connected by a curved lateral surface
right cone	a cone in which a perpendicular line drawn from the base to the tip (vertex) passes through the center of the base
sphere	a three-dimensional figure with all points the same distance from the center

Lesson 18.1 Describe and Analyze Cross Sections of Circular Solids

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can identify the shapes of cross sections of solids and solve problems involving the areas of cross sections.

Learning Objective

Identify and describe the two-dimensional figures resulting from horizontal and vertical cross-sections of solids.

Language Objective

Describe and analyze cross sections of solids comparing sides of cross sections to the radius, diameter, and height of solids.

Vocabulary

Review: pyramid, rectangular prism
New: cross section

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 18.2 Derive and Apply Formulas for Surface Areas of Cubes and Right Prisms

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can derive and apply the formulas for surface area of any right prism.

Learning Objective

Calculate the surface area of a right prism using the surface area formula.

Language Objective

Derive and explain how to use the surface area formula.

Vocabulary

Review: surface area

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 18.3 Derive and Apply a Formula for the Volume of a Right Prism

Connect Concepts and Skills –1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Model with mathematics.
• Use appropriate tools strategically.

I Can Objective

I can accurately apply the formula to find the volume of right prisms.

Learning Objective

Calculate the volume of a right prism using the volume formula.

Language Objective

Derive and explain how to use the volume formula for a right prism.

Lesson Materials

centimeter cubes, base-ten blocks;
grid paper (Teacher Resource Masters)

Lesson 18.4 Find Volume of Cylinders

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can find the volume of a cylinder or the dimensions of a cylinder given the volume.

Learning Objective

Develop and use the formula for the volume of a cylinder.

Language Objective

Use the terms *radius* and *height* to explain how to find the volume of a cylinder.

Vocabulary

New: cylinder

Lesson 18.5 Find Volume of Cones and Spheres

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can find the volume of a cone and of a sphere, and find the dimensions of a cone and of a sphere, given their volumes.

Learning Objective

Develop and use the formula for the volume of a cone and the volume of a sphere.

Language Objective

Use mathematical terminology to explain how to develop and use the formula for the volume of a cone and the volume of a sphere.

Vocabulary

New: right cone, sphere

Lesson 18.6 Solve Multi-Step Problems with Surface Area and Volume

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can solve multi-step surface area and volume problems involving real-world situations.

Learning Objective

Solve multi-step problems involving three-dimensional figures using formulas for surface area and volume.

Language Objective

Explain how to use the surface area formula to find volume and the volume formula to find surface area.

Unit 7: Probability

Unit 7 Project: Class Arcade
Unit 7 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 19: Understand and Apply Experimental Probability

Recommended Pacing with Assessments: 8 Days

Module 19 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students summarized numerical data sets in relation to their context. Students identified and represented proportional relationships.	Students conduct experiments to infer probabilities and rank outcomes by likelihood. Students represent sample spaces for compound events. Students design and use simulations to generate frequencies for simple and compound events. Students approximate the probability of a chance event by collecting data and predict the approximate relative frequency given the probability.	Students will understand and apply theoretical probability. Students will calculate theoretical probabilities of compound events and understand conditional probability. Students will use data from a sample survey to estimate a population mean or proportion. Students will develop a margin of error through the use of simulation models for random sampling.

Module 19 Vocabulary

event	an outcome or set of outcomes of an experiment or situation
complement of an event	the set of all outcomes that are not the event
compound event	an event made up of two or more simple events
experiment	in probability, any activity based on chance, such as tossing a coin
experimental probability	the ratio of the number of times an event occurs to the total number of trials, or times that the activity is performed
outcome	a possible result of a probability experiment
probability	a number from 0 to 1 (or 0% to 100%) that describes how likely an event is to occur
probability of an event	the ratio of the number of outcomes in the event to the total number of outcomes in the sample space
sample space	all possible outcomes of an experiment
simulation	a model of an experiment, often one that would be too difficult or too time-consuming to actually perform
trial	each repetition or observation of an experiment

Lesson 19.1 Understand Probability of an Event

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can describe the likelihood of an event.

Learning Objective

Describe the likelihood of an event in terms of a probability between 0 and 1.

Language Objective

Use probability, a number between 0 and 1 (or 0% and 100%), to describe the likelihood of an event.

Vocabulary

Review: event
New: experiment, outcome, probability, probability of an event, sample space, trial

Lesson Materials

number cubes, coins

Lesson 19.2 Find Experimental Probability of Simple Events

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

I Can Objective

I can find an experimental probability and its complement.

Learning Objective

Find the experimental probability of an event and its complement.

Language Objective

Conduct experiments to infer probabilities and describe outcomes by likelihood.

Vocabulary

New: complement of an event, experimental probability, simulation

Lesson Materials

paper cups, coins, number cubes; blank spinners (Teacher Resource Masters)

Lesson 19.3 Find Experimental Probability of Compound Events

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Design and use a simulation to generate frequencies for compound events.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can determine the experimental probability of compound events.

Learning Objective

Determine the experimental probability of compound events.

Language Objective

Use the terms *compound event* and *simulation* to explain the solutions to real-world probability problems.

Vocabulary

New: compound event

Lesson Materials

coins, number cubes

Lesson 19.4 Use Experimental Probability and Proportional Reasoning to Make Predictions

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can use proportional reasoning or percent expressions to make a prediction based on an experimental probability.

Learning Objective

Use experimental probability and proportional reasoning to make predictions about real-world scenarios.

Language Objective

Explain the meaning of proportions and percent equations and how to use them to predict the frequency of long-term events.

Unit 7: Probability

Unit 7 Project: Class Arcade
Unit 7 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 20: Understand and Apply Theoretical Probability

Recommended Pacing with Assessments and Performance Task: 9 Days

Module 20 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students approximated the probability of a chance event by collecting data and predicted the approximate relative frequency given the probability. Students developed a probability model by observing frequencies in chance process data. Students found the experimental probability of a compound event. Students developed an understanding of statistical variability.	Students approximate the probability of a chance event by collecting data. Students develop a uniform probability model by assigning equal probability to all outcomes. Students understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Students design and use a simulation to generate frequencies for compound events. Students solve multi-step problems posed with positive rational numbers.	Students will calculate probabilities of compound events and understand conditional probability. Students will investigate probability distributions. Students will investigate models of data for real-world events.

Module 20 Vocabulary

theoretical probability	the ratio of the number of possible outcomes in the event to the total number of possible outcomes in the sample space
tree diagram	a branching diagram that shows all possible combinations or outcomes of an event

Lesson 20.1 Find Theoretical Probability of Simple Events

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can find the theoretical probability of a simple event.

Learning Objective

Find the theoretical probability of simple events and compare theoretical probability to experimental probability.

Language Objective

Interpret the context of simple events and describe how to find the theoretical probability.

Vocabulary

New: theoretical probability

Lesson Materials

coins; blank spinners (Teacher Resource Masters)

Lesson 20.2 Find Theoretical Probability of Compound Events

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

Mathematical Practices and Processes

• Look for and make use of structure.

I Can Objective

I can find the theoretical probability of a compound event.

Learning Objective

Find and compare theoretical probabilities of compound events using a table, a tree diagram, and an organized list.

Language Objective

Interpret the context of compound events and describe how to find the theoretical probability of a compound event.

Vocabulary

Review: tree diagram

Lesson Materials

blank spinners (Teacher Resource Masters)

Lesson 20.3 Use Theoretical Probability and Proportional Reasoning

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probability of events.

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can use theoretical probability to make predictions about real-world situations.

Learning Objective

Use theoretical probability and proportional reasoning to make a prediction about a simple or compound event.

Language Objective

Make qualitative predictions about an event, and describe reasons for any discrepancies.

Lesson Materials

number cubes; blank spinners (Teacher Resource Masters)

Lesson 20.4 Conduct Simulations

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Design and use a simulation to generate frequencies for compound events.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can use a simulation to test the probability of simple and compound events.

Learning Objective

Design and perform a simulation to test the probability of a simple event or a compound event.

Language Objective

Interpret the context of a real-world scenario and explain how to determine the best simulation model to test the probability of a simple or compound event.

Lesson Materials

Digit Cards (Teacher Resource Masters)