HMH Into Math™ Accelerated 7

Unit 1: Ratios and Proportional Reasoning

Unit 1 Project: To Infinity (Almost) … and Beyond!
Unit 1 Learning Mindset Focus: Perseverance: Collects and Tries Multiple Strategies

Module 1: Identify and Represent Proportional Relationships

Recommended Pacing with Assessments: 13 Days

Module 1 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved problems involving unit pricing and speed. Students solved ratio and rate problems using tables of ratios, tape diagrams, double number line diagrams, and equations. Students used ratio and rate reasoning to solve real-world and mathematical problems.	Students identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions. Students represent proportional relationships by equations. Students solve problems of length and area using scale drawings.	Students will connect proportional relationships to linear functions, lines, linear equations, and graphs of linear functions. Students will compare two different proportional relationships represented in different ways. Students will understand similarity.

Module 1 Vocabulary

dimension	the length, width, or height of a figure
equation	a mathematical sentence that shows that two expressions are equivalent
ratio	a comparison of two quantities by division
reciprocal	one of two numbers whose product is 1
unit rate	a rate in which the second quantity in the comparison is one unit
constant of proportionality	a constant ratio of two variables related proportionally
proportional relationship	a relationship between two quantities in which the ratio of one quantity to the other quantity is constant
scale	the ratio between two sets of measurements
scale drawing	a drawing that uses a scale to make an object smaller than or larger than the real object

Lesson 1.1 Explore Relationships

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Recognize and represent proportional relationships between quantities.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can recognize when relationships presented in tables, diagrams, and verbal descriptions can be represented by a constant unit rate.

Learning Objective

Use patterns and unit rates to analyze and describe relationships.

Language Objective

Identify unit rates from verbal descriptions.

Vocabulary

Review: unit rate

Lesson Materials

two-color counters

Lesson 1.2 Recognize Proportional Relationships in Tables

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Represent proportional relationships by equations.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make sure of structure.

I Can Objective

I can identify proportional relationships in tables and equations, identify the constant of proportionality, and write the associated equation.

Learning Objective

Determine if a relationship represented in a table is proportional, identify the constant of proportionality, and write an equation in the form of $y = kx$.

Language Objective

Analyze and describe relationships in tables and identify and write equations for proportional relationships.

Vocabulary

Review: equation, ratio
New: constant of proportionality, proportional relationship

Lesson Materials

two-color counters

Lesson 1.3 Compute Unit Rates Involving Complex Fractions

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can compute unit rates associated with ratios of fractions.

Learning Objective

Use unit rates involving fractions to solve real-world problems.

Language Objective

Explain how to find and use unit rates involving fractions.

Vocabulary

Review: reciprocal

Lesson Materials

fraction strips

Lesson 1.4 Recognize Proportional Relationships in Graphs

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can decide whether a relationship shown in a graph is proportional and explain the connection between the constant of proportionality and the point $(1, r)$ on the graph.

Learning Objective

Students will identify the characteristics of a proportional relationship when graphed.

Language Objective

Explain how to determine whether a graph shows a proportional relationship, and if so, what the constant of proportionality is.

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 1.5 Use Proportional Relationships to Solve Rate Problems

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can identify the constant of proportionality and write an equation for a proportional relationship presented in various forms and use them to solve multi-step ratio problems.

Learning Objective

Use a proportional relationship to solve multi-step problems.

Language Objective

Explain how to use unit rates, tables, graphs, and equations to solve multi-step problems.

Lesson 1.6 Practice Proportional Reasoning with Scale Drawings

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can make scale drawings and use them to find actual dimensions.

Learning Objective

Use scale drawings to solve problems.

Language Objective

Explain how to use and interpret scale drawings.

Vocabulary

Review: dimension
New: scale, scale drawing

Unit 1: Ratios and Proportional Reasoning

Unit 1 Project: To Infinity (Almost) ... and Beyond!
Unit 1 Learning Mindset Focus: Perseverance: Collects and Tries Multiple Strategies

Module 2: Proportional Reasoning with Percents

Recommended Pacing with Assessments and Performance Task: 8 Days

Module 2 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students used ratio language to describe a ratio relationship between two quantities. Students defined unit rate. Students solved problems by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations, and found equivalent rates.	Students use proportional reasoning to calculate percent increase, percent decrease, markups, markdowns, taxes, gratuities, commissions, fees, simple interest, and the total value of an account. Students represent markups, markdowns, retail prices, discount prices, taxes, gratuity, total cost, commissions, and fees by equations of the form $y = kx$ by applying proportional reasoning. Students use proportional reasoning to assess the reasonableness of their answers. Students rewrite an expression in different forms to shed light on how the quantities in it are related.	Students will graph proportional relationships, interpreting the unit rate as the slope of the graph. Students will compare two different proportional relationships represented in different ways.

Module 2 Vocabulary

commission	a fee paid to a person for making a sale
fee	a fixed amount or a percent of an amount
gratuity	a tip, or monetary percentage that is given or paid in addition to the price of a service
markdown	the amount of decrease in a price
markup	the amount of increase in a price
percent change	the amount stated as a percent that a number increases or decreases
percent decrease	a percent change describing a decrease in a quantity
percent increase	a percent change describing an increase in a quantity

principal	the initial amount of money borrowed or saved
retail price	the amount an item is sold for after a company adds a markup or markdown
sales tax	a percent of the cost of an item that is charged by governments to raise money
simple interest	a fixed percent of the principal found using the formula $I = Prt$, where P represents the principal, r the rate of interest, and t the time
tip	another word for gratuity, a monetary percent that is given or paid in addition to the price of a service

Lesson 2.1 Percent Change

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can solve multi-step problems involving percent change.

Learning Objective

Use proportional reasoning to calculate percent increase or decrease.

Language Objective

Use the terms *percent increase* and *percent decrease* to explain the solutions to real-world problems.

Vocabulary

New: percent change, percent decrease, percent increase

Lesson 2.2 Markups and Discounts

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can calculate markups, markdowns, and retail prices and write equations for markup and markdown situations.

Learning Objective

Calculate markups, markdowns, retail prices, and discount prices, and represent them using equations of the form $y = kx$.

Language Objective

Use the terms *markup*, *markdown*, and *retail price* to explain the solutions to real-world problems.

Vocabulary

New: markdown, markup, retail price

Lesson 2.3 Taxes and Gratuities

Apply and Practice – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can find taxes, gratuities, and total costs by writing and using equations of the form $y = kx$, and assess the reasonableness of results.

Learning Objective

Represent taxes, gratuities, and total cost using equations in the form $y = kx$ by applying proportional reasoning. Use the equations to solve problems and assess reasonableness of their answers.

Language Objective

Use the terms *tax* and *gratuity* to explain the solutions to real-world problems.

Vocabulary

New: gratuity, sales tax, tip

Lesson 2.4 Commissions and Fees

Apply and Practice – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can calculate commissions, fees, and total earnings and assess the reasonableness of my results.

Learning Objective

Use proportional reasoning to find total earnings for someone earning a base salary plus a commission. Use proportional reasoning to find fees (including fees as percent and as a constant) and assess the reasonableness of their answers.

Language Objective

Use the terms *commission* and *fee* to explain the solutions to real-world problems.

Vocabulary

New: commission, fee

Lesson 2.5 Simple Interest

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

- Look for and make use of structure.
- Attend to precision.

I Can Objective

I can calculate simple interest and the total value of an account after any period of time. I understand and can apply the equation $I = Prt$.

Learning Objective

Use proportional reasoning to calculate simple interest, the total value of an account earning simple interest, and assess the reasonableness of their answers.

Language Objective

Use the term *simple interest* to explain the solutions to real-world problems.

Vocabulary

New: principal, simple interest

Unit 2: Number Systems and Operations

Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 3: Understand Addition and Subtraction of Rational Numbers

Recommended Pacing with Assessments: 7 Days

Module 3 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood positive and negative integers. Students added and subtracted positive numbers with a positive result. Students plotted numbers on a number line.	Students use a number line to add and subtract positive and negative integers and rational numbers. Students solve real-world problems involving addition and subtraction of positive and negative integers and of rational numbers.	Students will multiply and divide rational numbers. Students will solve real-world and mathematical problems involving the four operations with rational numbers.

Module 3 Vocabulary

degree	the unit of measure for angle or temperature
opposites	two numbers are opposites if, on a number line, they are the same distance from 0 but on opposite sides
Addition Property of Opposites	the property that states that the sum of a number and its opposite equals zero
additive inverse	the opposite of a number, such that the sum of a number and its additive inverse is 0

Lesson 3.1 Add or Subtract a Positive Integer on a Number Line

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

I Can Objective

I can use a number line to add and subtract positive integers.

Learning Objective

Use a number line to add and subtract positive integers.

Language Objective

Explain how to use a number line to add and subtract positive integers.

Vocabulary

Review: degree

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 3.2 Add or Subtract a Negative Integer on a Number Line

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.

I Can Objective

I can use a number line to add and subtract negative integers.

Learning Objective

Use a number line to add and subtract a negative integer and then assess their results for reasonableness.

Language Objective

Explain how to use a number line to add or subtract positive integers.

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 3.3 Use a Number Line to Add and Subtract Rational Numbers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Describe situations in which opposite quantities combine to make 0.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Attend to precision

I Can Objective

I can use a number line to add and subtract rational numbers.

Learning Objective

Use a number line to add and subtract rational numbers.

Language Objective

Explain how to use a number line to add and subtract rational numbers.

Vocabulary

Review: opposites

New: Addition Property of Opposites, additive inverse

Lesson Materials

number lines (Teacher Resource Masters)

Unit 2: Number Systems and Operations

Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 4: Fluency with Rational Number Operations

Recommended Pacing with Assessments: 11 Days

Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students performed operations with positive rational numbers. Students divided multi-digit numbers. Students interpreted positive and negative numbers as opposites on a number line. Students used ordering and absolute value of rational numbers. Students added, subtracted, multiplied, and divided integers.	Students add, subtract, multiply, and divide rational numbers. Students solve multi-step real-world and mathematical problems involving the strategic use of operations on rational numbers.	Students will solve equations involving rational numbers. Students will apply the properties of integer exponents to generate equivalent numerical expressions. Students will perform operations with numbers expressed in scientific notation.

Module 4 Vocabulary

absolute value	the distance of a number from zero on a number line; shown by $\vert\ \vert$
dividend	the number to be divided in a division problem
divisor	the number you are dividing by in a division problem
equivalent fractions	fractions that name the same amount or part
inverse operations	operations that undo each other; addition and subtraction or multiplication and division
quotient	the result when one number is divided by another
rational number	a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Lesson 4.1 Compute Sums of Rational Numbers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes

• Model with mathematics.

I Can Objective

I can compute sums of rational numbers with the same or different signs, and for real-world problems, I can interpret the results.

Learning Objective

Calculate the sum of rational numbers.

Language Objective

Explain how to add rational numbers when the sign of one or both addends is negative.

Vocabulary

Review: absolute value

Lesson Materials

two-color counters; number lines (Teacher Resource Masters)

Lesson 4.2 Compute Differences of Rational Numbers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes

• Model with mathematics.

I Can Objective

I can compute differences of rational numbers with the same or different signs, and for real-world problems, I can interpret the results.

Learning Objective

Calculate the difference of rational numbers.

Language Objective

Explain how to use algorithms to rewrite subtraction expressions as addition.

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 4.3 Understand and Compute Products and Quotients of Rational Numbers

Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.

Apply properties of operations as strategies to multiply and divide rational numbers.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Model with mathematics.

I Can Objective

I can apply the rules for multiplying and dividing rational numbers.

Learning Objective

Develop rules to find the products and quotients of rational numbers.

Language Objective

Explain how to use the rules for multiplying and dividing signed rational numbers to solve real-world and mathematical problems.

Vocabulary

Review: dividend, divisor, inverse operations, quotient

Lesson Materials

two-color counters; number lines (Teacher Resource Masters)

Lesson 4.4 Write Rational Numbers as Decimals

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.

Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can show that a number is rational by writing it as a ratio of integers, and I can convert a rational number to a decimal. I can show that $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$ for specific values of p and q.

Learning Objective

Express rational numbers as decimals.

Language Objective

Explain how to express rational numbers as decimals.

Vocabulary

Review: equivalent fractions, rational number

Lesson Materials

base-ten blocks, base-ten mat, fraction strips; Decimal Models, number lines (Teacher Resource Masters)

Lesson 4.5 Multiply and Divide Rational Numbers in Context

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can solve word problems that require multiplying and dividing rational numbers.

Learning Objective

Use products and quotients of rational numbers to solve problems.

Language Objective

Explain how to use products and quotients of rational numbers to solve problems.

Unit 2: Number Systems and Operations

Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 5: Applying Properties to Operations

Recommended Pacing with Assessments and Performance Task: 9 Days

Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students evaluated numerical expressions using parentheses, brackets, or braces. Students wrote simple expressions. Students interpreted numerical expressions.	Students apply the properties of operations to generate equivalent expressions. Students rewrite expressions to simplify solution processes. Students use variables to represent numbers and write expressions to solve problems. Students perform operations with algebraic expressions.	Students will apply properties of operations. Students will rewrite an expression in different forms. Students will use variables to represent quantities in real-world and mathematical problems. Students will construct simple equations and inequalities to solve problems. Students will expand expressions with rational coefficients.

Module 5 Vocabulary

equilateral triangle	a triangle in which all three sides are equal
quadrilateral	a four-sided figure

Lesson 5.1 Apply Properties to Multi-Step Problems with Rational Numbers

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Apply properties of operations as strategies to add and subtract rational numbers.

Apply properties of operations as strategies to multiply and divide rational numbers.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Attend to precision.

I Can Objective

I can apply properties of operations to evaluate multi-step expressions with positive and negative rational numbers.

Learning Objective

Use properties to solve multi-step problems involving positive and negative rational numbers.

Language Objective

Describe how to write and evaluate expressions involving 3 or more integers and signed rational numbers to represent real-world problems using properties, algorithms, and integer rules, converting between forms of rational numbers as appropriate.

Lesson 5.2 Solve Multi-Step Problems with Rational Numbers in Context

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Use appropriate tools strategically.
• Attend to precision.

I Can Objective

I can solve multi-step problems that involve rational numbers in different forms and multiple operations.

Learning Objective

Solve multi-step problems involving a combination of rational-number operations.

Language Objective

Explain how to solve multi-step problems and justify the reasonableness of answers.

Lesson 5.3 Add, Subtract, Factor, and Expand Algebraic Expressions

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can add, subtract, factor, and expand algebraic expressions with rational coefficients, and apply these skills to real-world problems.

Learning Objective

Add, subtract, factor, and expand linear expressions with rational coefficients.

Language Objective

Discuss how to write expressions to model real-world situations and then simplify those expressions by using the commutative property of addition and the distributive property.

Vocabulary

Review: equilateral triangle, quadrilateral

Lesson Materials

number lines (Teacher Resource Masters)

Unit 3: Equations and Inequalities in One Variable

Unit 3 Project: The Rhind Papyrus
Unit 3 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 6: Solve Linear Equations

Recommended Pacing with Assessments: 9 Days

Module 6 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students applied the properties of operations to generate equivalent expressions. Student solved real-world problems by writing and solving equations of the form $x + p = q$ and $px = q$, where p, q, and x are all non-negative rational numbers.	Students write and solve multi-step one-variable linear equations. Students interpret solutions of linear equations in context. Students explore and interpret equations. Students solve real-world problems involving multi-step linear equations in one variable. Students use supplementary, complementary, vertical, and adjacent angles in multi-step problems.	Students will write equations in one variable and use them to solve problems. Students will solve linear equations with coefficients represented by letters. Students will know and apply the properties of integer exponents. Students will use square and cube root symbols to solve equations. Student will evaluate square and cube roots.

Module 6 Vocabulary

coefficient	the number that is multiplied by the variable in an algebraic expression
common denominator	a denominator that is the same in two or more fractions
Distributive Property	For all real numbers a, b, and c, $a(b + c) = ab + ac$, and $a(b - c)= ab - ac$.
expression	a mathematical phrase that contains operations, numbers, and/or variables
isolate the variable	to get a variable alone on one side of an equation or inequality in order to solve the equation or inequality
like terms	terms having the same variable(s) raised to the same exponent
multiple	the product of any number and any nonzero whole number is a multiple of that number
solution of an equation	a value or values that make an equation true
adjacent angles	angles in the same plane that have a common vertex and a common side

complementary angles	angles whose measures have a sum of 90°
infinitely many solutions	occurs when every value of *x* creates a true mathematical statement
no solution	occurs when every value of *x* creates a false mathematical statement
supplementary angles	angles whose measures have a sum of 180°
vertical angles	a pair of opposite congruent angles formed by intersecting lines

Lesson 6.1 Write Two-Step Equations for Situations

Build Conceptual Understanding – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can write two-step equations for various situations.

Learning Objective

Represent a real-world situation with an equation.

Language Objective

Write equations of the form $px + q = r$ and $p(x + q) = r$ to represent real-world situations.

Vocabulary

Review: expression, like terms

Lesson Materials

algebra tiles, equation mat

Lesson 6.2 Apply Two-Step Equations to Solve Real-World Problems

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can apply two-step equations to solve a variety of problems.

Learning Objective

Solve real-world situations using an equation.

Language Objective

Explain how to solve real-world situations that can be modeled with equations.

Vocabulary

Review: solution of an equation

Lesson Materials

algebra tiles, equation mat; number line (Teacher Resource Masters)

Lesson 6.3 Solve Multi-step Linear Equations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can solve linear equations with integer and rational number coefficients.

Learning Objective

Use algebraic properties to solve one-variable linear equations.

Language Objective

Explain how to solve one-variable linear equations.

Vocabulary

Review: coefficient, common denominator, Distributive Property, isolate the variable, multiple

Lesson Materials

algebra tiles, equation mat

Lesson 6.4 Examine Special Cases

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.

I Can Objective

I can recognize linear equations that have no solution, one solution, or infinitely many solutions.

Learning Objective

Recognize and interpret linear equations that have no solution or infinitely many solutions.

Language Objective

Explain how to recognize linear equations that have no solution or infinitely many solutions.

Vocabulary

New: infinitely many solutions, no solution

Lesson Materials

algebra tiles, equation mat

Lesson 6.5 Apply Linear Equations

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Solve linear equations in one variable.

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

• Look for and make use of structure.
• Model with mathematics.

I Can Objective

I can solve equations and interpret solutions in context.

Learning Objective

Solve and apply linear equations in one variable.

Language Objective

Explain how to solve and apply linear equations in one variable.

Vocabulary

New: adjacent angles, complementary angles, supplementary angles, vertical angles

Unit 3: Equations and Inequalities in One Variable

Unit 3 Project: The Rhind Papyrus
Unit 3 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 7: Solve Problems Using Inequalities

Recommended Pacing with Assessments and Performance Task: 7 Days

Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote inequalities of the form $x > c$ or $x < c$ to represent constraints or conditions in a real-world or mathematical problem. Students solved equations as a process of answering a question. Students used variables to represent quantities in a real-world or mathematical problem, and constructed simple equations to solve problems by reasoning about the quantities.	Students solve one- and two-step inequalities for real-world problems. Students write and solve inequalities in the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Students graph the solution sets of inequalities and interpret them in the context of the problem.	Students will solve compound inequalities. Students will graph and solve inequalities in two variables.

Module 7 Vocabulary

inequality	a mathematical sentence that shows the relationship between quantities that are not equivalent
number line	a line used to plot real numbers, which include rational numbers and irrational numbers
solution of an inequality	a value or values that make an inequality true
rate of change	a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable

Lesson 7.1 Understand and Apply Properties to Solve One-Step Inequalities

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical Practices and Processes

- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can write and solve one-step inequalities.

Learning Objective

Apply properties to solve one-step inequalities.

Language Objective

Explain how to set up and solve one-step inequalities.

Vocabulary

Review: inequality, number line, solution of an inequality
New: rate of change

Lesson Materials

algebra tiles, equation mat; number lines (Teacher Resource Masters)

Lesson 7.2 Write Two-Step Inequalities for Situations

Connect Concepts and Skills – 1 Day

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can write two-step inequalities to solve problems.

Learning Objective

Write two-step inequalities to represent situations.

Language Objective

Use the terms *greater than*, *less than*, *greater than or equal to*, and *less than or equal to* in the context of inequalities to represent situations.

Lesson Materials

algebra tiles, equation mat; number lines (Teacher Resource Masters)

Lesson 7.3 Apply Two-Step Inequalities to Solve Problems

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can write and solve two-step inequalities two solve problems.

Learning Objective

Write and solve two-step inequalities to solve problems in context.

Language Objective

Write inequalities to represent situations and interpret the solutions in context.

Lesson Materials

algebra tiles, equation mat

Unit 4: Transform and Construct Geometric Figures

Unit 4 Project: A Puzzling Transformation
Unit 4 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 8: Transformations and Congruence

Recommended Pacing with Assessments: 11 Days

Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and drew geometric shapes with given conditions. Students used a ruler to measure length. Students used a protractor to measure angles. Students drew figures in the coordinate plane. Students understood and used properties of translations, reflections, and rotations.	Students explore and verify the properties of lines and angles in transformations. Students use coordinates to describe the effect of translations, reflections, and rotations. Students execute a sequence of transformations on a figure in the coordinate plane that results in a congruent figure.	Students will represent, compare, and recognize congruent figures using transformations. Students will understand and use translations, reflections, and rotations.

Module 8 Vocabulary

coordinate plane	a plane formed by the intersection of a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis
corresponding angles	for two lines intersected by a transversal, a pair of angles that lie on the same side of the transversal and on the same sides of each of the other two lines
origin	the point where the *x*-axis and *y*-axis intersect on the coordinate plane; (0, 0)
parallelogram	a quadrilateral with two pairs of parallel sides
quadrant	the *x*- and *y*-axes divide the coordinate plane into four regions. Each region is called a quadrant
vertex	on an angle or polygon, the point where two sides intersect; on a polyhedron, the intersection of three or more faces; on a cone or pyramid, the top point
***x*-axis**	the horizontal axis on a coordinate plane
***y*-axis**	the vertical axis on a coordinate plane
center of rotation	the point about which a figure is rotated
congruent	having the same size and shape; the symbol for congruent is ≅
image	a figure resulting from a transformation
line of reflection	the line across which a figure is reflected

preimage	the original figure in a transformation
prime notation	labeling images in a diagram by adding apostrophes to each letter
reflection	a transformation of a figure that flips the figure across a line
rotation	a transformation in which a figure is turned around a point
transformation	a change in the size or position of a figure
translation	a movement (slide) of a figure along a straight line

Lesson 8.1 Investigate Transformations

Build Conceptual Understanding – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations:

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can describe what happens to the sides and angles of a figure when it is transformed.

Learning Objective

Explore and observe the effects of rigid motions on a figure.

Language Objective

Explain how sliding, turning, and flipping a figure affects its shape, size, and direction the shape faces.

Vocabulary

New: transformation

Lesson Materials

ruler, protractor

Lesson 8.2 Explore Translations

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations:

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can translate figures, describe the translations using words and mapping notation, and determine an algebraic rule for translating a figure on a coordinate plane.

Learning Objective

Describe translations and their effects on a figure.

Language Objective

Explain how translations affect figures.

Vocabulary

Review: coordinate plane, vertex

New: image, preimage, prime notation, translation

Lesson Materials

protractor, ruler; grid paper (Teacher Resource Masters)

Lesson 8.3 Explore Reflections

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations.

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can reflect a figure over either axis in the coordinate plane and describe the reflection algebraically.

Learning Objective

Describe reflections and their effects on a figure.

Language Objective

Explain how to reflect figures on the coordinate plane, and describe the effects of a reflection algebraically.

Vocabulary

Review: corresponding angles, parallelogram, quadrant, *x*-axis, *y*-axis
New: line of reflection, reflection

Lesson Materials

ruler, protractor; Coordinate Plane (Teacher Resource Masters)

Lesson 8.4 Explore Rotations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations.

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can identify and perform rotations, and describe a rotation on a coordinate plane algebraically.

Learning Objective

Recognize and perform rotations. Describe rotations algebraically. Understand that rotating a figure produces an image that is congruent to the preimage.

Language Objective

Describe rotations and their effects on a figure.

Vocabulary

Review: origin
New: center of rotation, rotation

Lesson Materials

ruler, protractor; grid paper, Coordinate Plane (Teacher Resource Masters)

Lesson 8.5 Understand and Recognize Congruent Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

• Use appropriate tools strategically.
• Attend to precision.

I Can Objective

I can determine congruence by performing or describing a sequence of transformations that maps one figure onto another.

Learning Objective

Perform and describe sequences of transformations on figures.

Language Objective

Describe sequences of transformations on figures.

Vocabulary

New: congruent

Lesson Materials

ruler

Unit 4: Transform and Construct Geometric Figures

Unit 4 Project: A Puzzling Transformation
Unit 4 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 9: Draw and Analyze Two-Dimensional Figures

Recommended Pacing with Assessments: 7 Days

Module 9 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and drew lines of symmetry. Students found areas of figures by decomposing those figures into triangles. Students found volumes of right rectangular prisms. Students understood the concept of a ratio. Students used ratio and rate reasoning to solve real-world and mathematical problems. Students used angle relationships to solve problems.	Students draw and construct circles and other figures with given conditions using technology and freehand. Students analyze how many, if any, triangles can be made from given side lengths or angle measures. Students demonstrate the rule that the sum of the three angles of a triangle equals 180 degrees.	Students will use informal arguments to establish facts about the angle sum and exterior angles of triangles. Students will use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Students will compare two different proportional relationships represented in different ways. Students will understand similarity.

Module 9 Vocabulary

diameter	a line segment that passes through the center of a circle and has endpoints on the circle; or the length of that segment
radius	a line segment with one endpoint at the center of a circle and the other endpoint on the circle; or the length of that segment

Lesson 9.1 Draw Shapes with Given Conditions

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

• Use appropriate tools strategically.
• Look for and make use of structure.

I Can Objective

I can inscribe triangles in circles and draw geometric figures meeting given conditions.

Learning Objective

Draw and construct circles and other figures using technology and freehand with given conditions.

Language Objective

Describe figures using the words *circumference*, *radius*, *diameter*, *symmetry*, *parallel*, and *perpendicular*.

Vocabulary

New: diameter, radius

Lesson Materials

ruler, compass, protractor

Lesson 9.2 Draw and Construct Triangles Given Side Lengths

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Use appropriate tools strategically.

I Can Objective

I can determine whether three lengths could be side lengths of a triangle, and, given two side lengths, I can find the range of possible lengths for the third side.

Learning Objective

Determine how many triangles or quadrilaterals can be made given the side lengths: none, one, or many.

Language Objective

Understand and be able to explain whether you can form no triangle, one triangle, or many triangles from three given side lengths. Understand and be able to explain whether you can form no quadrilateral, one quadrilateral, or many quadrilaterals from four given side lengths.

Lesson Materials

ruler, compass

Lesson 9.3 Draw and Construct Triangles Given Angle Measures

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can use tools to construct triangles when given angle measures and determine if no triangle or many triangles can be formed.

Learning Objective

Determine how many triangles can be made given the angle measures: none, one, or many.

Language Objective

Describe angle measures and the relationship they form in a triangle.

Lesson Materials

ruler, protractor, compass; grid paper (Teacher Resource Masters)

Lesson 9.4 Draw and Analyze Shapes to Solve Problems

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

I Can Objective

I can draw and analyze shapes, including circles and triangles, to solve real-world problems.

Learning Objective

Draw, construct, and analyze two-dimensional figures to solve real-world problems.

Language Objective

Use the terms *center*, *diameter*, and *radius* in explaining how to construct and analyze circles.

Lesson Materials

ruler, protractor, compass; grid paper (Teacher Resource Masters)

Unit 4: Transform and Construct Geometric Figures

Unit 4 Project: A Puzzling Transformation
Unit 4 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 10: Transformations and Similarity

Recommended Pacing with Assessments and Performance Task: 9 Days

Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students drew polygons in the coordinate plane. Students drew geometric shapes with given conditions. Students solved problems involving scale drawings. Students used coordinates to find the length of a horizontal or vertical segment.	Students learn to describe the effects of dilations. Students dilate images on and off the coordinate plane. Students understand that dilations produce similar figures and the difference between similarity and congruence. Students describe a sequence of transformations that exhibits the similarity between two given figures. Students decide whether two given figures are similar.	Students will verify the properties of dilations. Students will understand dilations and scale factor. Students will prove all circles are similar. Students will use similarity to derive formulas. Students will establish the Angle-Angle Criterion. Students will solve problems and prove relationships in geometric figures.

Module 10 Vocabulary

enlargement	an increase in the size of all dimensions in the same proportion
center of dilation	the point of intersection of lines through each pair of corresponding vertices in a dilation
dilation	a transformation that enlarges or reduces a figure
scale factor	the ratio used to enlarge or reduce similar figures
similar	figures with the same shape but not necessarily the same size
reduction	a decrease in the size of all dimensions in the same proportion

Lesson 10.1 Investigate Reductions and Enlargements

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can identify and perform enlargements and reductions.

Learning Objective

Perform enlargements and reductions. Understand the image that is the result of enlarging or reducing a preimage is not congruent to the preimage.

Language Objective

Enlarge and reduce images on and off the coordinate plane and describe the effects of these transformations on the figure.

Vocabulary

Review: enlargement
New: reduction

Lesson Materials

ruler, protractor; grid paper (Teacher Resource Masters)

Lesson 10.2 Explore Dilations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can identify and perform dilations given a scale factor and center of dilation, perform a dilation on a coordinate plane, and identify an algebraic rule for the dilation.

Learning Objective

Describe and apply the properties of dilations. Understand and find the scale factor and center of dilation, both on and off the coordinate plane.

Language Objective

Dilate images on and off the coordinate plane and recognize that angle measures remain the same while side lengths must be proportional.

Vocabulary

New: center of dilation, dilation, scale factor

Lesson Materials

ruler, protractor; grid paper (Teacher Resource Masters)

Lesson 10.3 Understand and Recognize Similar Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can describe a sequence of transformations that exhibits the similarity between two figures.

Learning Objective

Recognize and draw similar figures using transformations.

Language Objective

Draw and identify similar figures and describe transformations that map one similar figure to another.

Vocabulary

New: similar

Lesson Materials

ruler, protractor

Unit 5: Similarity, Slope, and Linear Relationships

Unit 5 Project: Which Car Costs Less?
Unit 5 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 11: Angle Relationships

Recommended Pacing with Assessments: 8 Days

Module 11 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and used supplementary, complementary, vertical, and adjacent angles in multi-step problems. Students understood and described similar figures.	Students establish facts about the angle-measure sum and exterior angle of triangles. Students use angle measures to determine whether two triangles are similar. Students show that corresponding angles, alternate exterior angles, and alternate interior angles are congruent, and that same-side interior or exterior angles are supplementary. Students use these facts to find missing angles.	Students will prove theorems about triangles. Students will use triangles to solve problems. Students will work with similar figures. Students will prove theorems about lines and angles. Students will solve problems using theorems about lines and angles.

Module 11 Vocabulary

alternate exterior angles	for two lines intersected by a transversal, a pair of angles that lie on opposite sides of the transversal and outside the other two lines
alternate interior angles	for two lines intersected by a transversal, a pair of nonadjacent angles that lie on opposite sides of the transversal and between the other two lines
Angle-Angle Similarity Postulate	the postulate that states that two triangles are similar if they have two sets of corresponding angles that are congruent
corresponding angles	for two lines intersected by a transversal, a pair of angles that lie on the same side of the transversal and on the same sides of the other two lines
exterior angle	an angle formed by one side of a polygon and the extension of an adjacent side
Exterior Angle Theorem	the theorem that states that the measure of an exterior angle of a triangle is greater than either of the measures of the remote interior angles
remote interior angle	an interior angle of a polygon that is not adjacent to the exterior angle
same-side exterior angles	a pair of angles on the same side of a transversal but outside the parallel lines
same-side interior angles	a pair of angles on the same side of a transversal and between two lines intersected by the transversal

transversal	a line that intersects two or more lines
Triangle Sum Theorem	the theorem that states that the measures of the angles in a triangle add up to 180°

Lesson 11.1 Develop Angle Relationships for Triangles

Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.

I Can Objective

I can find an unknown angle measure in a triangle.

Learning Objective

Use angle relationships in triangles.

Language Objective

Use and describe angle relationships in triangles.

Vocabulary

New: exterior angle, Exterior Angle Theorem, remote interior angle, Triangle Sum Theorem

Lesson Materials

ruler, protractor; Polygons: Triangles (Teacher Resource Masters)

Lesson 11.2 Investigate Angle-Angle Similarity

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

I Can Objective

I can use angle-angle similarity to test triangles for similarity and find unknown angle measures.

Learning Objective

Identify whether two triangles are similar, given angle measures in the triangles. Find unknown angle measures in triangles known to be similar.

Language Objective

Explain how to identify two similar triangles based on the presence of two pairs of congruent angles.

Vocabulary

New: Angle-Angle Similarity Postulate

Lesson Materials

ruler, protractor

Lesson 11.3 Explore Parallel Lines Cut by a Transversal

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can identify the relationship between angle pairs as either supplementary or congruent.

Learning Objective

Find unknown angle measures when parallel lines are cut by a transversal.

Language Objective

Identify the angle pairs made when parallel lines are cut by a transversal and say whether the angles are congruent or supplementary.

Vocabulary

New: alternate exterior angles, alternate interior angles, corresponding angles, same-side exterior angles, same-side interior angle, transversal

Lesson Materials

ruler, protractor

Unit 5: Similarity, Slope, and Linear Relationships

Unit 5 Project: Which Car Costs Less?
Unit 5 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 12: Linear Relationships

Recommended Pacing with Assessments and Performance Task: 8 Days

Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved problems involving scale drawings. Students recognized and represented proportional relationships. Students understood the concept of a unit rate. Students analyzed relationships using graphs.	Students use similar triangles to understand slope. Students derive the equation $y = mx$. Students write an equation of a proportional relationship. Students interpret the meaning of the slope and y-intercept in a context. Students sketch, analyze and describe graphs of nonlinear relationships.	Students will create equations to represent relationships between quantities. Students will understand the concept of a function. Students will understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Students will interpret statements that use function notation in terms of a context. Students will compare properties of two functions.

Module 12 Vocabulary

unit rate	a rate in which the second quantity in the comparison is one unit
hypotenuse	in a right triangle, the side opposite the right angle
legs	in a right triangle, the sides that include the right angle; in an isosceles triangle, the pair of congruent sides
linear equation	an equation whose solutions form a straight line on a coordinate plane
linear relationship	a relationship whose graph is a straight line
nonlinear relationship	a relationship whose graph is not a straight line
rise	the vertical change when the slope of a line is expressed as the rise-over-run ratio
run	the horizontal change when the slope of a line is expressed as the rise-over-run ratio
slope	a measure of the steepness of a line on a graph: the rise divided by the run

slope-intercept form	a linear equation written in the form $y = mx + b$, where m represents slope and b represents the y-intercept
$\boldsymbol{y = mx + b}$	equation for a line that passes through the origin
y-intercept	the y-coordinate of the point where the graph of a line crosses the y-axis

Lesson 12.1 Explain Slope with Similar Triangles

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can determine the slope of a line and use it to find additional points on the line.

Learning Objective

Relate right triangles to the coordinates of a line going through the origin, and compare persistent features of the triangles to persistent features of the line.

Language Objective

Describe how to use similar right triangles to verify that the slope of a line is constant.

Vocabulary

Review: unit rate

New: hypotenuse, legs, rise, run, slope

Lesson Materials

protractor, ruler

Lesson 12.2 Derive $y = mx$

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can write the equation of a line given a graph or a table of values.

Learning Objective

Write the equation of a proportional relationship.

Language Objective

Explain how to identify the unit rate of change and write an equation that represents a proportional relationship.

Vocabulary

New: linear equation, $y = mx$

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 12.3 Derive and Interpret $y = mx + b$

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can derive the equation for a line in the form $y = mx + b$ given the slope of the line and a point.

Learning Objective

Write the equation of a line.

Language Objective

Explore lines to derive the equation $y = mx + b$, and be able to use the equation in all four quadrants to describe points on lines.

Vocabulary

New: slope-intercept form, y-intercept

Lesson Materials

coordinate grids

Lesson 12.4 Describe and Sketch Nonlinear Relationships

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can convert between a verbal description of a function and its graph, and between a graph and a verbal description of a relationship.

Learning Objective

Sketch and analyze a graph that exhibits qualitative features.

Language Objective

Sketch graphs given the description of the graph. Describe a relationship given its graph.

Vocabulary:

New: linear relationship, nonlinear relationship

Unit 6: Applications of Real Numbers and Exponents

Unit 6 Project: The Wheel of Theodorus
Unit 6 Learning Mindset Focus: Resilience: Adjusts to Change

Module 13: Real Numbers

Recommended Pacing with Assessments: 6 Days

Module 13 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood a rational number as a point on the number line. Students converted a rational number to a decimal. Students classified fractions and decimals as rational numbers. Students graphed rational numbers on a number line. Students wrote and evaluated numerical expressions involving whole-number exponents.	Students determine whether a number is rational or irrational. Students write rational numbers as decimals or fractions. Students evaluate square roots and cube roots and solve equations. Students identify decimal estimates of square roots and cube roots. Students compare numerical expressions involving roots. Students order a list of real numbers including rational and irrational numbers.	Students will generalize results from operations with rational and irrational numbers. Students will solve simple rational and radical equations in one variable. Students will understand notation for radicals in terms of rational exponents.

Module 13 Vocabulary

rational number	any number that can be expressed as a ratio of two integers
repeating decimal	a decimal in which one or more digits repeat infinitely
terminating decimal	a decimal number that ends, or terminates
cube root	the cube root of a positive number p is x when $x^3 = p$
irrational number	a number that cannot be expressed as a ratio of two integers or as a repeating or terminating decimal
perfect cube	a cube of a whole number
perfect square	a square of a whole number
principal square root	the nonnegative square root of a number
radical symbol	the symbol used to represent the nonnegative square root of a number

real number	a rational or irrational number
square root	the square root of a positive number p is x when $x^2 = p$

Lesson 13.1 Understand Rational and Irrational Numbers

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Use appropriate tools strategically.

I Can Objective

I can determine whether a number is rational and write a given rational number as a fraction.

Learning Objective

Determine if a number is rational.

Language Objective

Use the terms *rational*, *irrational*, *terminating decimal*, and *repeating decimal* to describe numbers.

Vocabulary

Review: rational number, repeating decimal, terminating decimal
New: irrational number

Lesson 13.2 Investigate Roots

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can evaluate square roots and cube roots.

Learning Objective

Evaluate square and cube roots.

Language Objective

Describe how to evaluate square and cube roots.

Vocabulary

New: cube root, perfect cube, perfect square, principal square root, radical symbol, square root

Lesson Materials

ruler, protractor

Lesson 13.3 Order Real Numbers

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Mathematical Practices and Processes

- Attend to precision.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can accurately order a list of real numbers containing fractions, decimals, and irrational numbers.

Learning Objective

Order a list of real numbers consisting of both rational and irrational numbers.

Language Objective

Use the terms *less than* or *greater than* to compare two real numbers.

Vocabulary

New: real numbers

Unit 6: Applications of Real Numbers and Exponents

Unit 6 Project: The Wheel of Theodorus
Unit 6 Learning Mindset Focus: Resilience: Adjusts to Change

Module 14: The Pythagorean Theorem

Recommended Pacing with Assessments: 6 Days

Module 14 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood irrational numbers and roots. Students drew geometric shapes with given conditions.	Students prove the Pythagorean Theorem and its converse. Students solve basic problems using the Pythagorean Theorem. Students apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Students apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.	Students will prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$. Students will derive the equation of a circle using the Pythagorean Theorem. Students will prove theorems about triangles. Students will prove the Pythagorean Theorem using triangle similarity. Students will use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Module 14 Vocabulary

cone	a three-dimensional figure with one vertex and one circular base
Pythagorean Theorem	In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
Pythagorean triple	a set of three positive integers a, b, and c such that $a^2 + b^2 = c^2$

Lesson 14.1 Prove the Pythagorean Theorem and Its Converse

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Explain a proof of the Pythagorean Theorem and its converse.

Mathematical Practices and Processes
- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.

I Can Objective
I can prove the Pythagorean Theorem and its converse.

Learning Objective
Prove and use the Pythagorean Theorem and its converse.

Language Objective
Use the terms *hypotenuse* and *legs* to describe parts of a right triangle. Explain the meaning of a Pythagorean triple and identify Pythagorean triples.

Use the terms converse, hypotenuse, and Pythagorean Theorem to prove that if $a^2 + b^2 = c^2$ is true for a triangle, then the triangle is a right triangle.

Vocabulary
New: Pythagorean Theorem, Pythagorean triple

Lesson Materials
ruler; grid paper, dot paper (Teacher Resource Masters)

Lesson 14.2 Apply the Pythagorean Theorem

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Mathematical Practices and Processes
- Attend to precision.
- Look for and make use of structure.

I Can Objective
I can apply the Pythagorean Theorem to solve real-life problems involving the legs and hypotenuse of a right triangle, including problems in three dimensions.

Learning Objective
Use the Pythagorean Theorem to solve real-world problems involving right triangles.

Language Objective
Classify the three sides of a right triangle as legs and the hypotenuse.

Vocabulary
Review: cone

Lesson 14.3 Apply the Pythagorean Theorem in the Coordinate Plane

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective
I can apply the Pythagorean Theorem to find the lengths of line segments on the coordinate plane, including line segments that are part of a composite figure.

Learning Objective
Use the Pythagorean Theorem to find the distance between any two points in the coordinate plane.

Language Objective
Explain how to use the Pythagorean Theorem to determine the distance between two points in a coordinate plane.

Lesson Materials
grid paper (Teacher Resource Masters); straightedge

Unit 6: Applications of Real Numbers and Exponents

Unit 6 Project: The Wheel of Theodorus
Unit 6 Learning Mindset Focus: Resilience: Adjusts to Change

Module 15: Exponents and Scientific Notation

Recommended Pacing with Assessments and Performance Task: 8 Days

Module 15 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote and evaluated numerical expressions involving whole-number exponents. Students applied the properties of operations to generate equivalent expressions. Students solved problems posed with positive and negative rational numbers in any form.	Students develop the properties of integer exponents. Students use the properties of integer exponents to simplify expressions and solve problems. Students translate between standard notation and scientific notation or vice versa. Students compare and compute numbers using scientific notation.	Students will choose and interpret units consistently in formulas. Students will explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents. Students will rewrite expressions involving radicals and rational exponents.

Module 15 Vocabulary

base	the number that is used as a factor when a number is raised to a power
exponent	the number that indicates how many times the base is used as a factor
power	a number produced by raising a base to an exponent
properties of exponents	properties for operations with exponents
scientific notation	a method of writing very large or very small numbers by using powers of 10
standard (or decimal) form of a number	way of writing a number by using digits

Lesson 15.1 Know and Apply Properties of Exponents

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can use properties of integer exponents to simplify expressions.

Learning Objective

Develop and use the properties of integer exponents.

Language Objective

Explain how to develop and use the properties of integer exponents to simplify expressions and solve problems.

Vocabulary

Review: base, exponent, power
New: properties of exponents

Lesson Materials

tape measure

Lesson 15.2 Understand Scientific Notation

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can use scientific notation to describe very large or very small quantities and to compare quantities.

Learning Objective

Express numbers using scientific notation.

Language Objective

Explain how to express numbers using scientific notation.

Vocabulary

New: scientific notation, standard form of a number

Lesson 15.3 Compute with Scientific Notation

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can compute with numbers in scientific notation and choose appropriate units for very large or small quantities.

Learning Objective

Compute with numbers written in scientific notation.

Language Objective

Explain how to compute with numbers written in scientific notation.

Unit 7: Area and Volume

Unit 7 Project: Buffon's Needle
Unit 7 Learning Mindset Focus: Perseverance: Learns Effectively

Module 16: Analyze Figures to Find Circumference and Area

Recommended Pacing with Assessments: 5 Days

Module 16 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students calculated circumference and area of circles. Students found surface area and volume of solids. Students found areas of triangles and special quadrilaterals. Students found the diameter and radius of circles.	Students know and use the formulas for the circumference and area of a circle to solve problems. Students solve multi-step real-life and mathematical problems involving positive rational numbers. Students calculate the area of a composite figure.	Students will derive the equation of a circle using the Pythagorean Theorem. Students will solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Students will calculate arc length, sector area, and analyze inscribed and circumscribed circles.

Module 16 Vocabulary

composite figure	a figure made up of simple geometric shapes
circumference	the distance around a circle for one rotation
pi (π)	the ratio of the circumference of a circle to the length of its diameter; $\pi \approx 3.14$ or $\frac{22}{7}$

Lesson 16.1 Derive and Apply Formulas for Circumference

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can find the circumference of a circle when I know either the radius or diameter.

Learning Objective

Derive and apply formulas for circumference.

Language Objective

Explain how to derive and apply formulas for circumference.

Vocabulary

New: circumference, pi (π)

Lesson Materials

compass, ruler

Lesson 16.2 Derive and Apply a Formula for the Area of a Circle

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can use formulas for the area and circumference of a circle to solve problems and can informally derive the relationship between the circumference and the area.

Learning Objective

Derive and apply formulas for the area of a circle.

Language Objective

Explain the meaning of the terms *radius*, *diameter*, *circumference*, and *area* of circles.

Lesson Materials

compass; grid paper (Teacher Resource Masters)

Lesson 16.3 Areas of Composite Figures

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Look for and make use of structure.
- Use appropriate tools strategically.

I Can Objective

I can break a composite figure into simple shapes and use area formulas to find its area.

Learning Objective

Use known formulas to calculate the areas of composite figures.

Language Objective

Explain how to calculate the area of a composite figure by breaking it into its component figures and adding their areas.

Vocabulary

Review: composite figure

Unit 7: Area and Volume

Unit 7 Project: Buffon's Needle
Unit 7 Learning Mindset Focus: Perseverance: Learns Effectively

Module 17: Analyze Surface Area and Volume

Recommended Pacing with Assessments and Performance Task: 10 Days

Module 17 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students described two-dimensional figures produced by slicing a three-dimensional circular solid. Students used nets to find the surface area of three-dimensional figures. Students found the volume of prisms. Students solved real-world and mathematical problems involving area of two-dimensional composite figures.	Students describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Students solve problems involving area, volume, and surface area. Students find volumes of spheres, cones, and cylinders and use them to solve problems.	Students will derive the equation of a circle using the Pythagorean Theorem. Students will give an informal argument using Cavalieri's principle for the formula for the volume of a sphere. Students will apply concepts of density based on area and volume in modeling situations.

Module 17 Vocabulary

pyramid	a polyhedron with a polygon base and triangular sides that all meet at a common vertex
rectangular prism	a polyhedron whose bases are rectangles and whose other faces are parallelograms
surface area	the sum of the areas of the faces, or surfaces, of a three-dimensional figure
cross section	the intersection of a three-dimensional figure and a plane
cylinder	a three-dimensional figure with two parallel, congruent circular bases connected by a curved lateral surface
right cone	a cone in which a perpendicular line drawn from the base to the tip (vertex) passes through the center of the base
sphere	a three-dimensional figure with all points the same distance from the center

Lesson 17.1 Describe and Analyze Cross Sections of Solids

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Mathematical Practices and Processes

• Look for and make use of structure.
rectangular pyramids.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can identify the shapes of cross sections of solids and solve problems involving the areas of cross sections.

Learning Objective

Identify and describe the two-dimensional figures resulting from horizontal and vertical cross-sections of solids.

Language Objective

Describe and analyze cross sections of solids comparing sides of cross sections to the radius, diameter, and height of solids.

Vocabulary

Review: pyramid, rectangular prism
New: cross section

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 17.2 Derive and Apply Formulas for Surface Areas of Cubes and Right Prisms

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Model with mathematics.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can derive and apply the formulas for surface area of any right prism.

Learning Objective

Calculate the surface area of a right prism using the surface area formula.

Language Objective

Derive and explain how to use the surface area formula.

Vocabulary

Review: surface area

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 17.3 Derive and Apply a Formula for the Volume of a Right Prism

Connect Concepts and Skills –1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Model with mathematics.
• Use appropriate tools strategically.

I Can Objective

I can accurately apply the formula to find the volume of right prisms.

Learning Objective

Calculate the volume of a right prism using the volume formula.

Language Objective

Derive and explain how to use the volume formula.

Lesson Materials

centimeter cubes, base-ten blocks;
grid paper (Teacher Resource Masters)

Lesson 17.4 Find Volume of Cylinders

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Attend to precision.
• Look for and make use of structure.

I Can Objective

I can find the volume of a cylinder or the dimensions of a cylinder given the volume.

Learning Objective

Develop and use the formula for the volume of a cylinder.

Language Objective

Use the terms *radius* and *height* to explain how to find the volume of a cylinder.

Vocabulary

New: cylinder

Lesson 17.5 Find Volume of Cones and Spheres

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can find the volume of a cone and of a sphere, and the dimensions of a cone and of a sphere given their volumes.

Learning Objective

Develop and use the formulas for the volume of a cone and the volume of a sphere.

Language Objective

Use mathematical terminology to explain how to develop and use the formula for the volume of a cone and the volume of a sphere.

Vocabulary

New: right cone, sphere

Lesson 17.6 Solve Multi-Step Problems with Surface Area and Volume

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can solve multi-step surface area and volume problems involving real-world situations.

Learning Objective

Solve multi-step problems involving three-dimensional figures using formulas for surface area and volume.

Language Objective

Explain how to use the surface area formula to find volume and the volume formula to find surface area.

Unit 8: Data Analysis and Sampling

Unit 8 Project: A Birthday Puzzle
Unit 8 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 18: Proportional Reasoning with Samples

Recommended Pacing with Assessments: 6 Days

Module 18 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students explored statistical data collection. Students learned how to choose a representative sample. Students analyzed and graphed statistical data. Students gave quantitative measures of center and variability.	Students understand how to select a representative sample. Students understand the ways that a sample could be biased. Students understand that proportional reasoning can be used to make inferences about a population from a random sample. Students use proportional reasoning to make inferences about a population. Students use data from a random sample to draw inferences about a population.	Students will compare data sets by measures of center. Students will compare data sets by measures of variability. Students will construct and interpret data in tables, including two-way frequency tables and two-way relative frequency tables. Students will use relative frequencies calculated for rows or columns to describe possible association between two variables.

Module 18 Vocabulary

bias	when a sample does not accurately represent the population
population	the entire group of objects or individuals considered for a survey
random sample	a sample in which each individual or object in the entire population has an equal chance of being selected
representative sample	a sample that has the same characteristics as the population
sample	a part of the population that is chosen to represent the entire group

Lesson 18.1 Understand Representative Samples

Build Conceptual Understanding – 1 Day

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can identify the population and sample for a given survey scenario and say whether a sample is random. I can determine whether a sample is likely to be representative of the population.

Learning Objective

Understand populations, random samples, and how to select a representative sample.

Language Objective

Properly use the terms *population* and *sample* when describing data collection.

Vocabulary

New: bias, population, random sample, representative sample, sample

Lesson Materials

Fraction Circles (Halves Through Sixths) (Teacher Resource Masters)

Lesson 18.2 Make Inferences from a Random Sample

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can use proportional reasoning to make inferences about populations based on the results of a random sample.

Learning Objective

Use a random sample to make inferences about a population.

Language Objective

Properly use the terms *population* and *sample* when making inferences.

Lesson Materials

Fraction Circles (Halves Through Sixths),
Grid of Quadrant I (Teacher Resource Masters)

Lesson 18.3 Make Inferences from Repeated Random Samples

Apply and Practice – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can use multiple random samples of the same size from a population to make inferences about a survey result.

Learning Objective

Understand that repeatedly sampling a population with the same size random sample will cause the data to vary.

Language Objective

Properly use the terms *population*, *sample*, and *representative sample* when discussing data from repeated samples.

Lesson Materials

Number Lines (Teacher Resource Masters)

Unit 8: Data Analysis and Sampling

Unit 8 Project: A Birthday Puzzle
Unit 8 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 19: Use Statistics and Graphs to Compare Data

Recommended Pacing with Assessments and Performance Task: 6 Days

Module 19 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students displayed data in dot plots and box plots. Students calculated five key values: minimum, lower quartile, median, upper quartile, and maximum. Students summarized and analyzed data using measures of center and measures of variability.	Students compare the shapes, centers, and spreads of data displayed in dot plots and box plots. Students draw conclusions about populations based on sample data displayed in dot plots and box plots. Students use measures of center and variability, including mean absolute deviation, for data from random samples to draw informal comparisons about the populations.	Students will construct two-way frequency tables and relative frequency tables. Students will use relative frequencies to describe an association between two variables.

Module 19 Vocabulary

box plot	a graph that shows how data are distributed by using the median, quartiles, least value, and greatest value
interquartile range	the difference between the upper and lower quartiles in a box plot
lower quartile	the median of the lower half of a set of data
mean	the sum of the items in a set of data divided by the number of items in the set; also called *average*
mean absolute deviation	the mean distance between each data value and the mean of the data set
median	the middle number, or the mean (average) of the two middle numbers, in an ordered set of data
range	the difference between the greatest and least values in a data set
upper quartile	the median of the upper half of a set of data

Lesson 19.1 Compare Center and Spread of Data Displayed in Dot Plots

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can compare two data sets displayed in dot plots and make inferences about two populations.

Learning Objective

Compare the center and spread of data displayed in dot plots.

Language Objective

Use the terms *center* and *spread* to describe key features of data sets displayed in dot plots.

Vocabulary

Review: mean, median, rang

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 19.2 Compare Center and Spread of Data Displayed in Box Plots

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective

I can compare two data sets displayed in box plots and make inferences about two populations.

Learning Objective

Compare data displayed in box plots, and use these comparisons to draw inferences about two populations.

Language Objective

Use the terms *box plot*, *range*, and *interquartile range* when comparing key features of data sets.

Vocabulary

Review: interquartile range, lower quartile, upper quartile

Lesson Materials

number lines (Teacher Resource Masters)

Lesson 19.3 Compare Means Using Mean Absolute Deviation and Repeated Sampling

Apply and Practice – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Mathematical Practices and Processes

• Attend to precision.

I Can Objective

I can use the means and MADS to assess the amount of visual overlap of two numerical data distributions.

Learning Objective

Use means and MADs to compare two populations.

Language Objective

Use the terms *mean absolute deviation* and *mean* to compare differences in center and spread of two populations.

Vocabulary

Review: mean absolute deviation

Lesson Materials

number lines (Teacher Resource Masters)

Unit 9: Probability

Unit 9 Project: Class Arcade
Unit 9 Learning Mindset Focus: Challenge-Seeking: Makes Plans to Meet Goals

Module 20: Understand and Apply Experimental Probability

Recommended Pacing with Assessments: 8 Days

Module 20 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students summarized numerical data sets in relation to their context. Students identified and represented proportional relationships.	Students conduct experiments to infer probabilities and rank outcomes by likelihood. Students represent sample spaces for compound events. Students design and use simulations to generate frequencies for simple and compound events. Students approximate the probability of a chance event by collecting data and predict the approximate relative frequency given the probability.	Students will understand and apply theoretical probability. Students will calculate theoretical probabilities of compound events and understand conditional probability. Students will use data from a sample survey to estimate a population mean or proportion. Students will develop a margin of error through the use of simulation models for random sampling.

Module 20 Vocabulary

complement of an event	the set of all outcomes that are not the event
compound event	an event made up of two or more simple events
event	an outcome or set of outcomes of an experiment or situation
experiment	in probability, any activity based on chance, such as tossing a coin
experimental probability	the ratio of the number of times an event occurs to the total number of trials, or times that the activity is performed
outcome	a possible result of a probability experiment
probability	a number from 0 to 1 (or 0% to 100%) that describes how likely an event is to occur
probability of an event	the ratio of the number of outcomes in the event to the total number of outcomes in the sample space
proportion	an equation that states that two ratios are equivalent
sample space	all possible outcomes of an experiment
simulation	a model of an experiment, often one that would be too difficult or too time-consuming to actually perform
trial	each repetition or observation of an experiment

Lesson 20.1 Understand Probability of an Event

Build Conceptual Understanding – 1 Day

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can describe the likelihood of an event.

Learning Objective

Describe the likelihood of an event in terms of a probability between 0 and 1.

Language Objective

Use probability, a number between 0 and 1 (or 0% and 100%), to describe the likelihood of an event.

Vocabulary

New: event, experiment, outcome, probability, probability of an event, sample space, trial

Lesson Materials

number cubes, coins

Lesson 20.2 Find Experimental Probability of Simple Events

Connect Concepts and Skills – 2 Days

Conceptual Build Conceptual Understanding	Conceptual and Procedural Connect Concepts and Skills	Procedural Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.

I Can Objective

I can find an experimental probability and its complement.

Learning Objective

Find the experimental probability of an event and its complement.

Language Objective

Conduct experiments to infer probabilities and describe outcomes by likelihood.

Vocabulary

New: complement of an event, experimental probability, simulation

Lesson Materials

paper cups, coins, number cubes; blank spinners (Teacher Resource Masters)

Lesson 20.3 Find Experimental Probability of Compound Events

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Design and use a simulation to generate frequencies for compound events.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can determine the experimental probability of compound events.

Learning Objective

Determine the experimental probability of compound events.

Language Objective

Use the terms *compound event* and *simulation* to explain the solutions to real-world probability problems.

Vocabulary

New: compound event

Lesson Materials

coins, number cubes

Lesson 20.4 Use Experimental Probability and Proportional Reasoning to Make Predictions

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can use proportional reasoning or percent expressions to make a prediction based on an experimental probability.

Learning Objective

Use experimental probability and proportional reasoning to make predictions about real-world scenarios.

Language Objective

Explain the meaning of proportions and percent equations and how to use them to predict the frequency of long-term events.

Unit 9: Probability

Unit 9 Project: Class Arcade
Unit 9 Learning Mindset Focus: Challenge-Seeking: Makes Plans to Meet Goals

Module 21: Understand and Apply Theoretical Probability

Recommended Pacing with Assessments and Performance Task: 9 Days

Module 21 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students approximated the probability of a chance event by collecting data and predicted the approximate relative frequency given the probability. Students developed a probability model and used it to find probabilities of events. Students found the experimental probability of a compound event. Students developed an understanding of statistical variability.	Students approximate the probability of a chance event by collecting data. Students develop a uniform probability model by assigning equal probability to all outcomes. Students understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Students design and use a simulation to generate frequencies for compound events. Students solve multi-step problems posed with positive rational numbers.	Students will calculate probabilities of compound events and understand conditional probability. Students will investigate probability distributions. Students will investigate models of data for real-world events.

Module 21 Vocabulary

complement	the set of all outcomes that are not the event
theoretical probability	the ratio of the number of possible outcomes in the event to the total number of possible outcomes in the sample space
tree diagram	a branching diagram that shows all possible combinations or outcomes of an event

Lesson 21.1 Find Theoretical Probability of Simple Events

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can find the theoretical probability of a simple event.

Learning Objective

Find the theoretical probability of simple events and compare theoretical probability to experimental probability.

Language Objective

Interpret the context of simple events and describe how to find the theoretical probability.

Vocabulary

New: theoretical probability

Lesson Materials

coins; blank spinners (Teacher Resource Masters)

Lesson 21.2 Find Theoretical Probability of Compound Events

Apply and Practice – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

Mathematical Practices and Processes

• Look for and make use of structure.

I Can Objective

I can find the theoretical probability of a compound event.

Learning Objective

Find and compare theoretical probabilities of compound events using a table, a tree diagram, and an organized list.

Language Objective

Interpret the context of compound events and describe how to find the theoretical probability of a compound event.

Vocabulary

New: tree diagram

Lesson Materials

blank spinners (Teacher Resource Masters)

Lesson 21.3 Use Theoretical Probability and Proportional Reasoning to Make Predictions

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probability of events.

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can use theoretical probability to make predictions about real-world situations.

Learning Objective

Use theoretical probability and proportional reasoning to make a qualitative prediction about a simple or compound event.

Language Objective

Make qualitative predictions about an event, and describe reasons for any discrepancies.

Vocabulary

Review: complement

Lesson Materials

number cubes; blank spinners (Teacher Resource Masters)

Lesson 21.4 Conduct Simulations

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Design and use a simulation to generate frequencies for compound events.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can use a simulation to test the probability of simple and compound events.

Learning Objective

Design and perform a simulation to test the probability of a simple event or a compound event.

Language Objective

Interpret the context of a real-world scenario and explain how to determine the best simulation model to test the probability of a simple or compound event.

Lesson Materials

Digit Cards (Teacher Resource Masters)