HMH into Math™ Grade 8

Unit 1: Transformational Geometry

Unit 1 Project: A Puzzling Transformation
Unit 1 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 1: Transformations and Congruence

Recommended Pacing with Assessments: 13 Days

Module 1 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and drew geometric shapes with given conditions. Students used a ruler to measure length. Students used a protractor to measure angles. Students drew figures in the coordinate plane. Students understood and used properties of translations, reflections, and rotations.	Students explore and verify the properties of lines and angles in transformations. Students use coordinates to describe the effect of translations, reflections, and rotations. Students execute a sequence of transformations on a figure in the coordinate plane that results in a congruent figure.	Students will represent, compare, and recognize congruent figures using transformations. Students will understand and use translations, reflections, and rotations.

Module 1 Vocabulary

coordinate plane	a plane formed by the intersection of a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis
line of reflection	a line that a figure is flipped across to create a mirror image of the original figure
origin	the point where the *x*-axis and *y*-axis intersect on the coordinate plane; (0, 0)
quadrant	the *x*- and *y*-axes divide the coordinate plane into four regions. Each region is called a quadrant
segment	a part of a line between two endpoints
vertex	on an angle or polygon, the point where two sides intersect; on a polyhedron, the intersection of three or more faces; on a cone or pyramid, the top point
***x*-axis**	the horizontal axis on a coordinate plane
***y*-axis**	the vertical axis on a coordinate plane
center of rotation	the point about which a figure is rotated
congruent	having the same size and shape

image	a figure resulting from a transformation
mapping notation	a rule used to express any type of transformation in the coordinate plane
preimage	the original figure in a transformation
prime notation	letters with apostrophes added that label images
reflection	a transformation of a figure that flips the figure across a line
rotation	a transformation in which a figure is turned around a point
transformation	a change in the size or position of a figure
translation	a movement (slide) of a figure along a straight line

Lesson 1.1 Investigate Transformations

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations:

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can describe what happens to the sides and angles of a figure when it is transformed.

Learning Objective

Explore and observe the effects of rigid motions on a figure.

Language Objective

Explain how sliding, turning, and flipping a figure affects its shape, size, and direction the shape faces.

Vocabulary

New: transformation

Lesson Materials

ruler, protractor

Lesson 1.2 Explore Translations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations:

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can translate figures, describe the translations using words and mapping notation, and determine an algebraic rule for translating a figure on a coordinate plane.

Learning Objective

Describe translations and their effects on a figure.

Language Objective

Explain how translations affect figures.

Vocabulary

Review: coordinate plane, segment, vertex
New: image, mapping notation, preimage, prime notation, translate

Lesson Materials

grid paper (Teacher Resource Masters), ruler, protractor

Lesson 1.3 Explore Reflections

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations.

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can reflect a figure over either axis in the coordinate plane and describe the reflection algebraically.

Learning Objective

Describe reflections and their effects on a figure.

Language Objective

Understand and explain how to reflect figures on the coordinate plane, and describe the effects of a reflection algebraically.

Vocabulary

Review: line of reflection, quadrant, *x*-axis, *y*-axis
New: reflection

Lesson Materials

ruler, protractor; grid paper (Teacher Resource Masters)

Lesson 1.4 Explore Rotations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Verify experimentally the properties of rotations, reflections, and translations.

Lines are taken to lines, and line segments to line segments of the same length.

Angles are taken to angles of the same measure.

Parallel lines are taken to parallel lines.

Describe the effect of dilations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Construct viable arguments and critique the reasoning of others.
- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can identify and perform rotations, and describe a rotation on a coordinate plane algebraically.

Learning Objective

Recognize and perform rotations. Describe rotations algebraically. Understand that rotating a figure produces an image that is congruent to the preimage.

Language Objective

Describe rotations and their effects on a figure.

Vocabulary

Review: origin
New: center of rotation, rotation

Lesson Materials

ruler, protractor; grid paper (Teacher Resource Masters)

Lesson 1.5 Understand and Recognize Congruent Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.

I Can Objective

I can determine congruence by performing or describing a sequence of transformations that maps one figure onto another.

Learning Objective

Perform and describe sequences of transformations on figures.

Language Objective

Describe sequences of transformations on figures.

Vocabulary

New: congruent

Lesson Materials

grid paper (Teacher Resource Masters), ruler

Unit 1: Number Systems and Operations

Unit 1 Project: A Puzzling Transformation
Unit 1 Learning Mindset Focus: Challenge-Seeking: Builds Confidence

Module 2: Transformations and Similarity

Recommended Pacing with Assessments and Performance Task: 11 Days

Module 2 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students drew polygons in the coordinate plane. Students drew geometric shapes with given conditions. Students solved problems involving scale drawings. Students used coordinates to find the length of a horizontal or vertical segment.	Students learn to describe the effects of dilations. Students dilate images on and off the coordinate plane. Students understand that dilations produce similar figures and the difference between similarity and congruence. Students describe a sequence of transformations that exhibits the similarity between two given figures. Students decide whether two given figures are similar.	Students will verify the properties of dilations. Students will understand dilations and scale factor. Students will prove all circles are similar. Students will use similarity to derive formulas. Students will establish the Angle-Angle Criterion. Students will solve problems and prove relationships in geometric figures.

Module 2 Vocabulary

reduction	a decrease in the size of all dimensions in the same proportion
center of dilation	the point of intersection of lines through each pair of corresponding vertices in a dilation
dilation	a transformation that enlarges or reduces a figure
scale factor	the ratio used to enlarge or reduce similar figures
similar	figures with the same shape but not necessarily the same size

Lesson 2.1 Investigate Reductions and Enlargements

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can identify and perform enlargements and reductions.

Learning Objective

Perform enlargements and reductions. Understand the image that is the result of enlarging or reducing a preimage is not congruent to the preimage.

Language Objective

Enlarge and reduce images on and off the coordinate plane and describe the effects of these transformations on the figure.

Vocabulary

Review: reduction

Lesson Materials

ruler, protractor; grid paper (Teacher Resource Masters)

Lesson 2.2 Explore Dilations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can identify and perform dilations given a scale factor and center of dilation, perform a dilation on a coordinate plane, and identify an algebraic rule for the dilation.

Learning Objective

Describe and apply the properties of dilations. Understand and find the scale factor and center of dilation, both on and off the coordinate plane.

Language Objective

Explain how to dilate images on and off the coordinate plane and why angle measures remain the same while side lengths must be proportional.

Vocabulary

New: center of dilation, dilation, scale factor

Lesson Materials

ruler, protractor; grid paper (Teacher Resource Masters)

Lesson 2.3 Understand and Recognize Similar Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.

I Can Objective

I can describe a sequence of transformations that exhibits the similarity between two figures.

Learning Objective

Recognize and draw similar figures using dilations.

Language Objective

Draw and identify similar figures and describe transformations that map one similar figure to another.

Vocabulary

New: similar

Lesson Materials

ruler, protractor

Unit 2: Linear Equations and Applications

Unit 2 Project: All in the Music
Unit 2 Learning Mindset Focus: Perseverance: Checks for Understanding

Module 3: Solve Linear Equations

Recommended Pacing with Assessments: 9 Days

Module 3 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote and solved single-step one variable linear equations. Students used variables to represent quantities. Students used supplementary, complementary, vertical, and adjacent angles in multistep problems.	Students write and solve multi-step one-variable linear equations. Students interpret solutions of linear equations in context. Students explore and interpret equations with no solutions, infinitely many solutions, and one solution. Students solve real-world problems involving multistep linear equations in one variable.	Students will write equations in one variable and use them to solve problems. Students will explain each step in solving a simple equation. Students will solve linear equations with coefficients represented by letters.

Module 3 Vocabulary

coefficient	the number that is multiplied by the variable in an algebraic expression
Distributive Property	for all real numbers a, b, and c, $a(b + c) = ab + ac$, and $a(b - c) = ab - ac$
expression	a mathematical phrase that contains operations, numbers, and/or variables
isolate the variable	to get a variable alone on one side of an equation or inequality in order to solve the equation or inequality
least common denominator	the least common multiple of two or more denominators
like terms	terms that have the same variable raised to the same exponents
substitute	to replace a variable with a number or another expression in an algebraic expression
infinitely many solutions	occurs if the graphs of two linear equations overlap and therefore intersect at infinitely many points
no solution	occurs when a system of two equations has graphs that never intersect because lines are parallel

Lesson 3.1 Solve Multi-step Linear Equations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can solve linear equations with integer and rational number coefficients.

Learning Objective

Use algebraic properties to solve one-variable linear equations.

Language Objective

Explain how to solve one-variable linear equations.

Vocabulary

Review: coefficient, Distributive Property, isolate the variable, least common denominator, like terms

Lesson Materials

algebra tiles, equation mat

Lesson 3.2 Examine Special Cases

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can recognize linear equations that have no solution, one solution, or infinitely many solutions. I can determine if all the solutions of an equation with infinitely many solutions make sense in a real-world context. I can interpret the meaning of an equation with no solution in a real-world context.

Learning Objective

Recognize and interpret linear equations that have no solution or infinitely many solutions.

Language Objective

Recognize linear equations that have no solution, one solution, or infinitely many solutions.

Vocabulary

Review: substitute
New: infinitely many solutions, no solution

Lesson Materials

algebra tiles, equation mat

Lesson 3.3 Apply Linear Equations

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve linear equations in one variable.

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can solve linear equations and interpret solutions in context.

Learning Objective

Solve and apply linear equations in one variable.

Language Objective

Construct and solve multi-step linear equations to solve real-world problems.

Vocabulary

Review: expression

Unit 2: Linear Equations and Applications

Unit 2 Project: All in the Music
Unit 2 Learning Mindset Focus: Perseverance: Checks for Understanding

Module 4: Angle Relationships

Recommended Pacing with Assessments and Performance Task: 11 Days

Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and used supplementary, complementary, vertical, and adjacent angles in multi-step problems. Students understood and described similar figures.	Students establish facts about the angle sum and exterior angle of triangles. Students use angle measures to determine whether two triangles are similar. Students show that corresponding angles, alternate exterior angles, and alternate interior angles are congruent, and that same-side interior or exterior angles are supplementary. Students use these facts to find missing angles.	Students will prove theorems about triangles. Students will use triangles to solve problems. Students will work with similar figures. Students will prove theorems about lines and angles. Students will solve problems using theorems about lines and angles.

Module 4 Vocabulary

interior angle	angles on the inner sides of two lines cut by a transversal
alternate exterior angles	for two lines intersected by a transversal, a pair of angles that lie on opposite sides of the transversal and outside the other two lines
alternate interior angles	for two lines intersected by a transversal, a pair of nonadjacent angles that lie on opposite sides of the transversal and between the other two lines
Angle-Angle Similarity Postulate	two triangles are similar if they have two sets of corresponding angles that are congruent
corresponding angles	for two lines intersected by a transversal, a pair of angles that lie on the same side of the transversal and on the same sides of the other two lines
exterior angle	an angle formed by one side of a polygon and the extension of an adjacent side
Exterior Angle Theorem	the measure of an exterior angle of a triangle is greater than either of the measures of the remote interior angles
remote interior angle	an interior angle of a polygon that is not adjacent to the exterior angle
same-side exterior angles	a pair of angles on the same side of a transversal but outside the parallel lines

same-side interior angles	a pair of angles on the same side of a transversal and between two lines intersected by the transversal
transversal	a line that intersects two or more lines
Triangle Sum Theorem	the theorem that states that the measures of the angles in a triangle add up to 180°

Lesson 4.1 Develop Angle Relationships for Triangles

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can find an unknown angle measure in a triangle.

Learning Objective

Use angle relationships in triangles.

Language Objective

Describe angle relationships in triangles.

Vocabulary

Review: interior angle
New: exterior angle, Exterior Angle Theorem, remote interior angle, Triangle Sum Theorem

Lesson Materials

ruler, protractor

Lesson 4.2 Investigate Angle-Angle Similarity

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

I Can Objective

I can use Angle-Angle similarity to test triangles for similarity and find missing angle measures.

Learning Objective

Identify whether two triangles are similar, given angle measures in the triangles.
The students will be able to find missing angle measures in triangles known to be similar.

Language Objective

Explain how to identify two similar triangles based on the presence of two pairs of congruent angles.

Vocabulary

New: Angle-Angle Similarity Postulate

Lesson Materials

ruler, protractor

Lesson 4.3 Explore Parallel Lines Cut by a Transversal

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can identify the relationship between angle pairs as either supplementary or congruent.

Learning Objective

Find missing angle measures when parallel lines are cut by a transversal.

Language Objective

Identify the angle pairs made when parallel lines are cut by a transversal and say whether the angles are congruent or supplementary.

Vocabulary

New: alternate exterior angles, alternate interior angles, corresponding angles, same-side exterior angles, same-side interior angle, transversal

Lesson Materials

ruler, protractor

Unit 3: Relationships and Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 5: Proportional Relationships

Recommended Pacing with Assessments: 11 Days

Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved problems involving scale drawings. Students understood and used similar triangles. Students recognized and represented proportional relationships. Students understood the concept of a unit rate. Students analyzed relationships using graphs.	Students derive the equation $y = mx$. Students write an equation of a proportional relationship. Students graph proportional relationships. Students identify and use unit rates. Students determine whether a graph should be discrete or continuous. Students compare proportional relationships presented in different representations.	Students will write equations in two variables. Students will understand the concept of a function. Students will understand the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Students will interpret statements that use function notation in terms of a context. Students will compare properties of two functions.

Module 5 Vocabulary

continuous graph	a graph made up of connected lines or curves
discrete graph	a graph made up of unconnected points
hypotenuse	in a right triangle, the side opposite the right angle
legs	in a right triangle, the sides that include the right angle; in an isosceles triangle, the pair of congruent sides
linear equation	an equation whose solutions form a straight line on a coordinate plane
rise	the vertical change when the slope of a line is expressed as the rise-over-run ratio
run	the horizontal change when the slope of a line is expressed as the rise-over-run ratio
slope	a measure of the steepness of a line on a graph: the rise divided by the run
unit rate	a rate in which the second quantity in the comparison is one unit

Lesson 5.1 Explain Slope with Similar Triangles

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can determine the slope of a line and use it to find additional points on the line.

Learning Objective

Relate right triangles to the coordinates of a line going through the origin, and compare persistent features of the triangles to persistent features of the line.

Language Objective

Describe how to use similar right triangles to verify that the slope of a line is constant.

Vocabulary

New: hypotenuse, legs, rise, run, slope, unit rate

Lesson Materials

protractor, ruler

Lesson 5.2 Derive $y = mx$

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Use similar triangles to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Mathematical Practices and Processes

- Model with mathematics.
- Attend to precision.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can write the equation of a line given a graph or a table of values.

Learning Objective

Write the equation of a proportional relationship.

Language Objective

Explain how to identify the unit rate of change and write an equation that represents a proportional relationship.

Vocabulary

New: linear equation

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 5.3 Interpret and Graph Proportional Relationships

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can graph proportional relationships from a table or equation, calculate the unit rate, and determine whether the graph should be continuous or discrete.

Learning Objective

Graph proportional relationships. Interpret unit rate as the slope of the graph of a proportional relationship.

Language Objective

Explain how to find the unit rate of a proportional relationship from graphs and tables and how to determine whether a graph should be continuous or discrete based on the situation it represents.

Vocabulary

New: discrete graph, continuous graph

Lesson Materials

Grid of Quadrant 1 (Teacher Resource Masters), ruler

Lesson 5.4 Compare Proportional Relationships

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can identify and compare proportional relationships presented in different ways.

Learning Objective

Demonstrate and interpret proportional relationships between quantities.

Language Objective

Use the terms *proportional* and *unit rate* to explain solutions to problems that involve proportional relationships.

Unit 3: Relationships and Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 6: Understand and Analyze Functions

Recommended Pacing with Assessments: 15 Days

Module 6 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students derived $y = mx$. Students recognized, represented, and compared proportional relationships. Students analyzed the relationship between dependent and independent variables. Students interpreted unit rate as slope. Students solved problems involving unit pricing and speed. Students understood and graphed functions.	Students learn to identify functions and give examples of functions that are not linear. Students use functions to solve real-world problems. Student derive and interpret $y = mx + b$. Students interpret the meaning of the slope and y-intercept in a context. Students construct a function. Students compare functions. Students sketch, analyze, and describe a graph that exhibits the qualitative features of a function.	Students will understand functions and use function notation. Students will relate the domain of a function to its graph. Students will graph linear functions and show intercepts. Students will write a function in different forms to reveal and explain different properties. Students will compare properties of two functions.

Module 6 Vocabulary

input	the value substituted into an expression or function
output	the value that results from the substitution of a given input into an expression or function
domain	the set of all possible input values of a function
function	an input-output relationship that has exactly one output for each input
linear function	a function whose graph is a straight line
nonlinear function	a function whose graph is not a straight line
range	the set of all possible output values of a function

relation	a set of ordered pairs
slope-intercept form	a linear equation written in the form $y = mx + b$, where m represents slope and b represents the y-intercept
vertical line test	a test used to determine whether a relation is a function; if any vertical line crosses the graph of a relation more than once, the relation is not a function
***y*-intercept**	the y-coordinate of the point where the graph of a line crosses the y-axis

Lesson 6.1 Understand and Graph Functions

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can graph a function given a table, and identify a function given a table or a graph.

Learning Objective

Students will visually display a relationship between two variables.

Language Objective

Explain domain and range, and differentiate functions from non-functions given data or a graph.

Vocabulary

Review: input, output
New: domain, function, range, relation, vertical line test

Lesson 6.2 Derive $y = mx + b$

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can derive the equation for a line in the form $y = mx + b$ given the slope of the line and a point.

Learning Objective

Students will write the equation of a linear function.

Language Objective

Explain how the equation $y = mx + b$ describes a function with slope m and y-intercept b.

Vocabulary

New: linear function, nonlinear function, slope-intercept form, y-intercept

Lesson 6.3 Interpret Rate of Change and Initial Value

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can find and interpret initial value and rate of change.

Learning Objective

Interpret the slope and y-intercept of a line.

Language Objective

Explain the connection between slope, rate of change, and m; and the connection between y-intercept, initial value, and b.

Lesson Materials

Grid of Quadrant 1 (Teacher Resource Masters)

Lesson 6.4 Construct Functions

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Practices and Processes

- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can construct functions based on verbal descriptions, tables, and graphs.

Learning Objective

Students will construct a function to model a linear relationship.

Language Objective

Explain how to determine if a function is discrete or continuous.

Lesson 6.5 Compare Functions

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can compare functions presented in equations, tables, graphs, and verbal descriptions.

Learning Objective

Use tables, graphs, and equations to compare functions.

Language Objective

Model functions based on verbal descriptions and be able to describe functions verbally.

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 6.6 Describe and Sketch Nonlinear Functions

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can convert between a verbal description of a function and its graph, and between a graph and a verbal description of a function.

Learning Objective

Students will sketch and analyze a graph that exhibits the qualitative features of a function.

Language Objective

Describe how to sketch a graph given a description of its function. Describe a function given its graph.

Unit 3: Relationships and Functions

Unit 3 Project: Which Car Costs Less?
Unit 3 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 7: Systems of Linear Equations

Recommended Pacing with Assessments and Performance Task: 17 Days

Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved real-world problems by graphing points in all four quadrants. Students understood solving an equation as a process of answering a question. Students wrote and solved one-variable equations. Students used variables to represent quantities in a real-world problem.	Students use graphing to determine the approximate solution to a system. Students solve systems of linear equations by graphing, substitution, and elimination. Students recognize and interpret graphic and algebraic results of solving a system with no solution or infinitely many solutions. Students use systems of two linear equations to represent real-world problems.	Students will solve systems of linear equations exactly and approximately. Students will prove elimination produces a system with the same solutions. Students will represent constraints by systems of equations. Students will interpret solutions as viable or non-viable in a modeling context.

Module 7 Vocabulary

infinitely many solutions	occurs if the graphs of two linear equations overlap and therefore intersect at infinitely many points
no solution	occurs when a system of two equations has graphs that never intersect because lines are parallel
substitute	to replace a variable with a number or another expression in an algebraic expression
elimination	algebraic process of eliminating a variable in a system of equations by combining the equations through addition
solution of a system of equations	a set of values that make all equations in a system true
system of equations	a set of two or more equations that contain two or more variables

Lesson 7.1 Represent Systems by Graphing

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Look for and make use of structure.

I Can Objective

I can graph a pair of linear equations and draw a conclusion from the graph.

Learning Objective

Interpret the graphical representation of two linear equations.

Language Objective

Describe the relationships in a graphical representation of two linear equations.

Lesson Materials

Grid of Quadrant 1 (Teacher Resource Masters)

Lesson 7.2 Solve Systems by Graphing

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Attend to precision.

I Can Objective

I can solve a system of linear equations by graphing and check that my solution is correct.

Learning Objective

Solve a system of two linear equations by graphing.

Language Objective

Describe how to use graphing to solve a system of two linear equations.

Vocabulary

New: solution of a system of equations, system of equations

Lesson Materials

Coordinate Plane (Teacher Resource Masters), ruler

Lesson 7.3 Solve Systems by Substitution

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Mathematical Practices and Processes

- Model with mathematics.
- Use appropriate tools strategically.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can solve systems of equations by substitution.

Learning Objective

Use substitution to solve a system of two linear equations.

Language Objective

Explain how to solve a system of two linear equations using substitution.

Vocabulary

Review: substitute

Lesson Materials

algebra tiles

Lesson 7.4 Solve Systems by Elimination

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can solve a system of linear equations by elimination and check my solution.

Learning Objective

Use elimination to solve a system of two linear equations.

Language Objective

Explain how a variable is eliminated by multiplying one equation by a factor and then adding the equations to solve the system.

Vocabulary

New: elimination

Lesson Materials

Coordinate Plane (Teacher Resource Masters)

Lesson 7.5 Examine Special Systems

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

I Can Objective

I can identify the number of solutions to a system of linear equations in any form.

Learning Objective

Recognize and interpret systems of two linear equations that have no solution or infinitely many solutions.

Language Objective

Relate the solutions of a system of equations to the intersections of lines on the coordinate plane.

Vocabulary

Review: infinitely many solutions, no solution

Lesson Materials

Coordinate Plane (Teacher Resource Masters)

Lesson 7.6 Apply Systems of Equations

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Analyze and solve pairs of simultaneous linear equations.

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.

I Can Objective

I can write and solve a system of equations to solve a real-world problem.

Learning Objective

Use systems of two linear equations to solve real-world problems.

Language Objective

Read and interpret a description of a context and translate this into a system of linear equations to solve real-world problems.

Unit 4: Statistics and Probability

Unit 4 Project: Altitude or Latitude?
Unit 4 Learning Mindset Focus: Challenge-Seeking: Makes Decisions

Module 8: Scatter Plots

Recommended Pacing with Assessments: 9 Days

Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students displayed and summarized numerical data in plots on a number line in context. Students understood that statistics can be used to gain information about a population. Students informally assessed the degree of visual overlap of two numerical data distributions. Students summarized numerical data sets in relation to their context.	Students recognize that lines can be used to model the pattern shown in a scatter plot. Students sketch a linear model for a data set displayed in a scatter plot. Students understand the influence of outliers on the trend line of a data set. Students compare linear models to determine which is the best fit for given data. Students interpret rate of change and initial value of a linear function.	Students will distinguish between correlation and causation and evaluate reports. Students will compute and interpret the correlation coefficient and interpret slope in context. Students will understand statistics as a process for making inferences.

Module 8 Vocabulary

association	the description of the relationship between two data sets
cluster	set of closely grouped data
linear association	when data points roughly lie along a line
negative association	two data sets have a negative association if one set of data values increases while the other decreases
no association	two data sets have no association when there is no relationship between their data values
nonlinear association	an association between two variables in which the data do not have a linear trend
outlier	a value much greater or much less than the others in a data set
positive association	two data sets have a positive association when their data values increase or decrease together
scatter plot	a graph with points plotted to show a possible relationship between two sets of data
trend line	a line on a scatter plot that helps show the association between data sets more clearly

Lesson 8.1 Construct Scatter Plots and Examine Association

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.

I Can Objective

I can construct a scatter plot, determining whether an association is positive or negative, strong or weak, and nonlinear or linear. I can recognize outliers and clusters.

Learning Objective

Display and analyze data with two variables.

Language Objective

Use the adjectives *positive*, *negative*, *strong*, and *weak* to describe the correlations between two variables shown in a scatter plot.

Vocabulary

New: association, cluster, linear association, negative association, no association, nonlinear association, outlier, positive association, scatter plot

Lesson Materials

grid paper (Teacher Resource Masters)

Lesson 8.2 Draw and Analyze Trend Lines

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can draw a trend line and informally assess a trend line by judging the closeness of the data points to the line.

Learning Objective

Use trend lines to describe a linear relationship between two variables.

Language Objective

Explain how to use a line to model the pattern shown by a data set.

Vocabulary

New: trend line

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 8.3 Interpret Linear Data in Context

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can determine the equation of a line to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Learning Objective

Use scatter plots and trend lines to interpret linear data in context.

Language Objective

Explain how to use a scatter plot and a trend line to determine reasonable values for data in context.

Lesson Materials

grid paper (Teacher Resource Masters)

Unit 4: Statistics and Probability

Unit 4 Project: Altitude or Latitude?
Unit 4 Learning Mindset Focus: Challenge-Seeking: Makes Decisions

Module 9: Two-Way Tables

Recommended Pacing with Assessments and Performance Task: 11 Days

Module 9 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students recognized and understood that the probability of a chance event expresses the likelihood of the event occurring. Students approximated probability by collecting data and observing its long-run relative frequency.	Students construct and interpret two-way frequency tables. Students determine whether there is an association between events. Students construct and interpret two-way relative frequency tables. Students calculate and interpret conditional relative frequencies.	Students will understand characteristics of independent events. Students will understand, recognize, and explain the concept of conditional probability. Students will determine if two events are independent.

Module 9 Vocabulary

conditional relative frequency	the ratio of a joint relative frequency to a related marginal relative frequency in a two-way table
joint relative frequency	the frequency in a particular category divided by the total number of data values
marginal relative frequency	the sum of the joint relative frequencies in a row or column of a two-way table
relative frequency	the frequency of a specific data value divided by the total number of data values in the set
two-way relative frequency table	a two-way table that displays relative frequencies
two-way table	a table that displays two-variable data by organizing it into rows and columns

Lesson 9.1 Construct and Interpret Two-Way Frequency Tables

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can construct a two-way table summarizing data, complete a table given partial data, and interpret data to determine whether there is an association between two variables.

Learning Objective

Interpret data by constructing two-way frequency tables.

Language Objective

Describe and justify conclusions derived from constructing and interpreting two-way frequency tables.

Vocabulary

New: two-way table

Lesson 9.2 Construct Two-Way Relative Frequency Tables

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can construct a two-way relative frequency table, complete a relative frequency table given partial data, and interpret joint and marginal relative frequencies.

Learning Objective

Construct two-way relative frequency tables.

Language Objective

Use terminology such as *joint relative frequency* and *marginal relative frequency* to explain how to construct and analyze two-way relative frequency tables.

Vocabulary

New: joint relative frequency, marginal relative frequency, relative frequency, two-way relative frequency table

Lesson 9.3 Interpret Two-Way Relative Frequency Tables

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can calculate conditional relative frequencies to determine whether there is an association between two variables and compare likelihoods from a table.

Learning Objective

Interpret and analyze data using two-way relative frequency tables.

Language Objective

Use the term *conditional relative frequency* to interpret and analyze data in two-way relative frequency tables.

Vocabulary

New: conditional relative frequency

Unit 5: Real Numbers and the Pythagorean Theorem

Unit 5 Project: The Wheel of Theodorus
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 10: Real Numbers

Recommended Pacing with Assessments: 9 Days

Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood a rational number as a point on the number line. Students converted a rational number to a decimal. Students wrote and solved one-step linear equations. Students graphed rational numbers on a number line.	Students determine whether a number is rational or irrational. Students write rational numbers as decimals or fractions. Students evaluate square roots and cube roots and solve equations. Students identify decimal estimates of square roots and cube roots. Students compare numerical expressions involving roots. Students order a list of real numbers including rational and irrational numbers.	Students will generalize results from operations with rational and irrational numbers. Students will solve simple rational and radical equations in one variable. Students will understand notation for radicals in terms of rational exponents.

Module 10 Vocabulary

circumference	the distance around a circle
cubes	a number raised to the third power
diameter	a line segment that passes through the center of a circle and has endpoints on the circle; or the length of that segment
greatest common factor	the largest common factor of two or more given numbers
number line	a line used to plot real numbers including integers, rational numbers, and irrational numbers
numerator	the top number of a fraction that tells how many parts of a whole are being considered
power	a number produced by raising a base to an exponent
rational number	any number that can be expressed as a ratio of two integers

cube root	a number, written as $\sqrt[3]{x}$, whose cube is x
irrational number	a number that cannot be expressed as a ratio of two integers or as a repeating or terminating decimal
perfect cube	a cube of a whole number
perfect square	a square of a whole number
pi	the ratio of the circumference of a circle to the length of its diameter; $\pi \approx 3.14$ or $\frac{22}{7}$
principal square root	the positive square root
radical symbol	the symbol $\sqrt{}$ used to represent the nonnegative square root of a number
real number	a rational or irrational number
repeating decimal	a decimal in which one or more digits repeat
square root	a number that is multiplied by itself to form a product is called a square root of that product
terminating decimal	a decimal number that ends, or terminates

Lesson 10.1 Understand Rational and Irrational Numbers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can determine whether a number is rational and write a given rational number as a fraction.

Learning Objective

Determine if a number is rational.

Language Objective

Use the terms *rational*, *irrational*, *terminating decimal*, and *repeating decimal* to describe numbers.

Vocabulary

Review: circumference, diameter, greatest common factor, numerator, power, rational number

New: irrational number, pi, repeating decimal, terminating decimal

Lesson 10.2 Investigate Roots

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Mathematical Practices and Processes

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

I Can Objective

I can evaluate square roots and cube roots.

Learning Objective

Evaluate square and cube roots.

Language Objective

Describe how to evaluate square and cube roots.

Vocabulary

Review: cubes
New: cube root, perfect cube, perfect square, principal square root, radical symbol, square root

Lesson Materials

ruler, protractor

Lesson 10.3 Order Real Numbers

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Mathematical Practices and Processes

- Look for and express regularity in repeated reasoning.
- Attend to precision.

I Can Objective

I can accurately order a list of real numbers containing fractions, decimals, and irrational numbers.

Learning Objective

Order a list of real numbers consisting of both rational and irrational numbers.

Language Objective

Use the terms *less than* or *greater than* to compare two real numbers.

Vocabulary

Review: number line
New: real number

Unit 5: Real Numbers and the Pythagorean Theorem

Unit 5 Project: The Wheel of Theodorus
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 11: The Pythagorean Theorem

Recommended Pacing with Assessments and Performance Task: 13 Days

Module 11 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood irrational numbers and roots. Students solved real-world and mathematical problems involving area, volume, and surface area.	Students prove the Pythagorean Theorem and its converse. Students solve basic problems using the Pythagorean Theorem. Students apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Students apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.	Students will prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$. Students will derive the equation of a circle using the Pythagorean Theorem. Students will prove theorems about triangles. Students will prove the Pythagorean Theorem using triangle similarity. Students will use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Module 11 Vocabulary

height	the perpendicular distance from the base to the opposite vertex or side
radius	a line segment with one endpoint at the center of the circle and the other endpoint on the circle, or the length of that segment
converse	a version of a theorem that has the hypothesis and the conclusion reversed
Pythagorean Theorem	In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
Pythagorean triple	a set of three positive integers a, b, and c such that $a^2 + b^2 = c^2$

Lesson 11.1 Prove the Pythagorean Theorem

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Explain a proof of the Pythagorean Theorem and its converse.

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can prove the Pythagorean Theorem, use the Pythagorean Theorem to find missing side lengths of right triangles, and identify a Pythagorean triple.

Learning Objective

Prove and use the Pythagorean Theorem.

Language Objective

Use the terms *hypotenuse* and *legs* to describe parts of a right triangle. Explain the meaning of a Pythagorean triple.

Vocabulary

New: Pythagorean Theorem, Pythagorean triple

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 11.2 Prove the Converse of the Pythagorean Theorem

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Explain a proof of the Pythagorean Theorem and its converse.

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can prove the converse of the Pythagorean Theorem and use it to determine if a triangle is a right triangle, and I can determine whether three side lengths form a Pythagorean triple.

Learning Objective

Prove and apply the Pythagorean Theorem and its converse.

Language Objective

Use the terms *converse*, *Pythagorean Theorem*, and *hypotenuse* to explain that if $a^2 + b^2 = c^2$ is true for a triangle, then the triangle is a right triangle.

Vocabulary

New: converse

Lesson Materials

ruler; grid paper (Teacher Resource Masters)

Lesson 11.3 Apply the Pythagorean Theorem

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Mathematical Practices and Processes

- Attend to precision.
- Construct viable arguments and critique the reasoning of others.

I Can Objective

I can apply the Pythagorean Theorem to solve real-life problems involving the legs and hypotenuse of a right triangle, including problems in three dimensions.

Learning Objective

Use the Pythagorean Theorem to solve real-world problems involving right triangles.

Language Objective

Classify the three sides of a right triangle as legs and the hypotenuse.

Vocabulary

Review: height, radius

Lesson 11.4 Apply the Pythagorean Theorem in the Coordinate Plane

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Mathematical Practices and Processes

- Look for and make use of structure.

I Can Objective

I can apply the Pythagorean Theorem to find the lengths of line segments on the coordinate plane, including line segments that are part of a composite figure.

Learning Objective

Use the Pythagorean Theorem to determine distance between any two points in the coordinate plane.

Language Objective

Explain how to use the Pythagorean Theorem to determine the distance between two points in a coordinate plane.

Lesson Materials

grid paper (Teacher Resource Masters)

Unit 6: Exponents, Scientific Notation, and Volume

Unit 6 Project: The Large and the Small of It
Unit 6 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 12: Exponents and Scientific Notation

Recommended Pacing with Assessments: 9 Days

Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote and evaluated numerical expressions involving whole-number exponents. Students used whole-number exponents to denote powers of 10. Students solved problems posed with positive and negative rational numbers in any form. Students knew and applied the properties of integer exponents.	Students develop the properties of integer exponents. Students use the properties of integer exponents to simplify expressions and solve problems. Students translate between standard notation and scientific notation or vice versa. Students compare and compute numbers using scientific notation.	Students will explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents. Students will rewrite expressions involving radicals and rational exponents. Students will choose and interpret units consistently in formulas.

Module 12 Vocabulary

base	the number that is used as a factor when a number is raised to a power
exponent	the number that indicates how many times the base is used as a factor
properties of exponents	properties for operations with exponents
scientific notation	a method of writing very large or very small numbers by using powers of 10
standard (or decimal) form	way of writing a number by using digits

Lesson 12.1 Know and Apply Properties of Exponents

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can use properties of integer exponents to simplify expressions.

Learning Objective

Develop and use the properties of integer exponents.

Language Objective

Explain how to use the properties of integer exponents to simplify expressions and solve problems.

Vocabulary

Review: base, exponent
New: properties of exponents

Lesson 12.2 Understand Scientific Notation

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can use scientific notation to describe very large or very small quantities and to compare quantities.

Learning Objective

Express numbers using scientific notation.

Language Objective

Explain how to express numbers using scientific notation.

Vocabulary

New: scientific notation, standard (or decimal) form

Lesson 12.3 Compute with Scientific Notation

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can compute with scientific notation, choose appropriate units for real-world quantities, and convert values in scientific notation from one unit to another.

Learning Objective

Compute with numbers written in scientific notation.

Language Objective

Explain how to compute with numbers written in scientific notation.

Unit 6: Exponents, Scientific Notation, and Volume

Unit 6 Project: The Large and the Small of It
Unit 6 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 13: Volume

Recommended Pacing with Assessments and Performance Task: 13 Days

Module 13 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students recognized volume as an attribute of solid figures and understood concepts of volume measurement. Students found volume of right rectangular prisms. Students measured volume by counting unit cubes. Student solved problems involving volume.	Students use the formulas for the volumes of cylinders, cones, and spheres. Students develop the formula for the volume of a cylinder. Students develop the formula for the volume of a cone. Students develop the formula for the volume of a sphere. Students use volume formulas to solve real–world problems.	Students will give an informal argument for the formula for the volume of cylinders and cones. Students will give an informal argument using Cavalieri's principle for the formula for the volume of a sphere. Students will use volume formulas to solve problems. Students will apply concepts of density based on area and volume in modeling situations.

Module 13 Vocabulary

cylinder	a three-dimensional figure with two parallel, congruent circular bases connected by a curved lateral surface
slant height	the distance from the vertex of a right cone to a point on the edge of the base
sphere	a three-dimensional figure with all points the same distance from the center

Lesson 13.1 Find Volume of Cylinders

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can find the volume of a cylinder or the dimensions of a cylinder given the volume.

Learning Objective

Develop and use the formula for the volume of a cylinder.

Language Objective

Use the terms *radius* and *height* to explain how to find the volume of a cylinder.

Vocabulary

Review: cylinder

Lesson 13.2 Find Volume of Cones

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Look for and make use of structure.

I Can Objective

I can find the volume of a cone and the dimensions of a cone given its volume.

Learning Objective

Develop and use the formula for the volume of a cone.

Language Objective

Use the terms *radius*, *height*, and *slant height* to explain how to find the volume of a cone.

Vocabulary

New: slant height

Lesson 13.3 Find Volume of Spheres

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices and Processes

- Attend to precision.
- Look for and make use of structure.

I Can Objective

I can find the volume of a sphere and the dimensions of a sphere given its volume.

Learning Objective

Develop and use the formula for the volume of a sphere.

Language Objective

Use mathematical terminology to explain how to develop and use the formula for the volume of a sphere.

Vocabulary

New: sphere

Lesson Materials

spherical objects, ruler

Lesson 13.4 Apply Volume

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Model with mathematics.

I Can Objective

I can use the formulas for the volume of cones, cylinders, and spheres to solve real-world problems.

Learning Objective

Use volume formulas to solve problems involving cylinders, cones, and spheres.

Language Objective

Use the terms *cylinder*, *cone*, *sphere*, and *volume* to explain the solutions to real-world problems.