Unit 1: Proportional Relationships

Unit 1 Project: To Infinity (Almost) … and Beyond!
Unit 1 Learning Mindset Focus: Perseverance: Collects and Tries Multiple Strategies

Module 1: Identify and Represent Proportional Relationships

Recommended Pacing with Assessments: 14 Days

Module 1 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved problems involving unit pricing and speed. Students solved ratio and rate problems using tables of ratios, tape diagrams, double number line diagrams, and equations. Students used ratio and rate reasoning to solve real-world and mathematical problems.	Students identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions. Students represent proportional relationships by equations. Students solve problems of length and area using scale drawings.	Students will connect proportional relationships to linear functions, lines, linear equations, and graphs of linear functions. Students will compare two different proportional relationships represented in different ways. Students will understand similarity.

Module 1 Vocabulary

dimension	the length, width, or height of a figure
equation	a mathematical sentence that shows that two expressions are equivalent
ratio	a comparison of two quantities by division
reciprocal	one of two numbers whose product is 1
unit rate	a rate in which the second quantity in the comparison is one unit
constant of proportionality	a constant ratio of two variables related proportionally
proportional relationship	a relationship between two quantities in which the ratio of one quantity to the other quantity is constant
scale	the ratio between two sets of measurements
scale drawing	a drawing that uses a scale to make an object smaller than or larger than the real object

Lesson 1.1 Explore Relationships

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Recognize and represent proportional relationships between quantities.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective

I can recognize when relationships presented in tables, diagrams, and verbal descriptions can be represented by a constant unit rate.

Learning Objective

Use patterns and unit rates to analyze and describe relationships.

Language Objective

Identify unit rates from verbal descriptions.

Vocabulary

Review: unit rate

Lesson Materials

two-color counters

Lesson 1.2 Recognize Proportional Relationships in Tables

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	**Connect Concepts and Skills**	Apply and Practice

Mathematics Standards

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Represent proportional relationships by equations.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.
• Look for and make sure of structure.

I Can Objective

I can identify proportional relationships in tables and equations, identify the constant of proportionality, and write the associated equation.

Learning Objective

Determine if a relationship represented in a table is proportional, identify the constant of proportionality, and write an equation in the form of $y = kx$.

Language Objective

Analyze and describe relationships in tables and identify and write equations for proportional relationships.

Vocabulary

Review: equation, ratio
New: constant of proportionality, proportional relationship

Lesson Materials

two-color counters

Lesson 1.3 Compute Unit Rates Involving Fractions
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Model with mathematics.
• Attend to precision.

I Can Objective
I can compute unit rates associated with ratios of fractions.

Learning Objective
Use unit rates involving fractions to solve real-world problems.

Language Objective
Explain how to find and use unit rates involving fractions.

Vocabulary
Review: reciprocal

Lesson Materials
fraction strips

Lesson 1.4 Recognize Proportional Relationships in Graphs
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Model with mathematics.
• Look for and make use of structure.

I Can Objective
I can decide whether a relationship shown in a graph is proportional and explain the connection between the constant of proportionality and the point (1, r) on the graph.

Learning Objective
Students will identify the characteristics of a proportional relationship when graphed.

Language Objective
Explain how to determine whether a graph shows a proportional relationship, and if so, what the constant of proportionality is.

Lesson Materials
grid paper (Teacher Resource Masters)

Lesson 1.5 Use Proportional Relationships to Solve Rate Problems

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Use proportional relationships to solve multi-step ratio and percent problems.

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.

I Can Objective

I can identify the constant of proportionality and write an equation for a proportional relationship presented in various forms and use them to solve multi-step ratio problems.

Learning Objective

Use a proportional relationship to solve multi-step problems.

Language Objective

Explain how to use unit rates, tables, graphs, and equations to solve multi-step problems.

Lesson 1.6 Practice Proportional Reasoning with Scale Drawings

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.

I Can Objective

I can make scale drawings and use them to find actual dimensions.

Learning Objective

Use scale drawings to solve problems.

Language Objective

Explain how to use and interpret scale drawings.

Vocabulary

Review: dimension
New: scale, scale drawing

Unit 1: Proportional Relationships

Unit 1 Project: To Infinity (Almost) … and Beyond!
Unit 1 Learning Mindset Focus: Perseverance: Collects and Tries Multiple Strategies

Module 2: Proportional Reasoning with Percents

Recommended Pacing with Assessments and Performance Task: 15 Days

Module 2 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students used ratio language to describe a ratio relationship between two quantities. Students defined unit rate. Students solved problems by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations, and found equivalent rates.	Students use proportional reasoning to calculate percent increase, percent decrease, markups, markdowns, taxes, gratuities, commissions, fees, simple interest, and the total value of an account. Students represent markups, markdowns, retail prices, discount prices, taxes, gratuity, total cost, commissions, and fees by equations of the form $y = kx$ by applying proportional reasoning. Students use proportional reasoning to assess the reasonableness of their answers.	Students will graph proportional relationships, interpreting the unit rate as the slope of the graph. Students will compare two different proportional relationships represented in different ways.

Module 2 Vocabulary

fee	a fixed amount or a percent of an amount
commission	a fee paid to a person for making a sale
commission rate	the fee paid to a person who makes a sale expressed as a percent of the selling price
gratuity	a tip, or monetary percentage that is given or paid in addition to the price of a service
markdown	the amount of decrease in a price
markup	the amount of increase in a price
percent change	the amount stated as a percent that a number increases or decreases
percent decrease	a percent change describing a decrease in a quantity
percent increase	a percent change describing an increase in a quantity

principal	the initial amount of money borrowed or saved
retail price	the amount an item is sold for after a company adds a markup or markdown
sales tax	a percent of the cost of an item that is charged by governments to raise money
simple interest	a fixed percent of the principal found using the formula $I = Prt$, where P represents the principal, r the rate of interest, and t the time
tip	another word for gratuity, a monetary percent that is given or paid in addition to the price of a service

Lesson 2.1 Percent Change
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Model with mathematics.
• Look for and make use of structure.

I Can Objective
I can solve multi-step problems involving percent change.

Learning Objective
Use proportional reasoning to calculate percent increase or decrease.

Language Objective
Use the terms *percent increase* and *percent decrease* to explain the solutions to real-world problems.

Vocabulary
New: percent change, percent decrease, percent increase

Lesson 2.2 Markups and Discounts

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use proportional relationships to solve multi-step ratio and percent problems.

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Mathematical Practices and Processes
• Model with mathematics.
• Attend to precision.

I Can Objective
I can calculate markups and markdowns.

Learning Objective
Calculate markups, markdowns, retail prices, and discount prices, and represent them using equations of the form $y = kx$.

Language Objective
Use the terms *markup*, *markdown*, and *retail price* to explain the solutions to real-world problems.

Vocabulary
Review: cost
New: markdown, markup, retail price

Lesson 2.3 Taxes and Gratuities

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Model with mathematics.
• Look for and make use of structure.

I Can Objective
I can use equations to represent the total cost of items with taxes and gratuities.

Learning Objective
Represent taxes, gratuities, and total cost using equations in the form $y = kx$ by applying proportional reasoning. Use the equations to solve problems and assess reasonableness of their answers.

Language Objective
Use the terms *tax* and *gratuity* to explain the solutions to real-world problems.

Vocabulary
New: gratuity, sales tax, tip

Lesson 2.4 Commissions and Fees
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Model with mathematics.
• Attend to precision.

I Can Objective
I can find the total earnings of a commission-based job.

Learning Objective
Use proportional reasoning to find total earnings for someone earning a base salary plus a commission. Use proportional reasoning to find fees (including fees as percent and as a constant) and assess the reasonableness of their answers.

Language Objective
Use the terms *commission* and *fee* to explain the solutions to real-world problems.

Vocabulary
Review: fee
New: commission, commission rate

Lesson 2.5 Simple Interest
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Model with mathematics.
• Attend to precision.
• Look for and make use of structure.

I Can Objective
I can calculate simple interest and the total value of an account after any period of time. I understand and can apply the equation $I = Prt$.

Learning Objective
Use proportional reasoning to calculate simple interest, the total value of an account earning simple interest, and assess the reasonableness of their answers.

Language Objective
Use the term *simple interest* to explain the solutions to real-world problems.

Vocabulary
New: principal, simple interest

Unit 2: Rational Number Operations

Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 3: Understand Addition and Subtraction of Rational Numbers

Recommended Pacing with Assessments: 9 Days

Module 3 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students understood positive and negative integers. Students added and subtracted positive numbers with a positive result. Students plotted numbers on a number line.	Students use a number line to add and subtract positive and negative integers and rational numbers. Students solve real-world problems involving addition and subtraction of positive and negative integers and of rational numbers.	Students will multiply and divide rational numbers. Students will solve real-world and mathematical problems involving the four operations with rational numbers.

Module 3 Vocabulary

Addition Property of Opposites	the property that states that the sum of a number and its opposite equals zero
additive inverse	the opposite of a number, such that the sum of a number and its additive inverse is 0

Lesson 3.1 Add or Subtract a Positive Integer on a Number Line
Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Use appropriate tools strategically.

I Can Objective
I can add or subtract positive integers on a number line.

Learning Objective
Use a number line to add and subtract positive integers.

Language Objective
Use a number line to add and subtract positive integers.

Lesson Materials
number lines (Teacher Resource Masters)

Lesson 3.2 Add or Subtract a Negative Integer on a Number Line
Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Mathematical Practices and Processes
• Construct viable arguments and critique the reasoning of others.
• Use appropriate tools strategically.

I Can Objective
I can add or subtract negative integers on a number line.

Learning Objective
Use a number line to add or subtract a negative integer and then assess their results for reasonableness.

Language Objective
Use a number line to add or subtract a negative integer and then access their results for reasonableness.

Lesson Materials
number lines

Lesson 3.3 Use a Number Line to Add and Subtract Rational Numbers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Describe situations in which opposite quantities combine to make 0.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Attend to precision

I Can Objective

I can use a number line to add or subtract rational numbers.

Learning Objective

Use a number line to add and subtract rational numbers.

Language Objective

Use a number line to add and subtract rational numbers.

Vocabulary

New: Addition Property of Opposites, additive inverse

Lesson Materials

number lines

Unit 2: Rational Number Operations
Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 4: Add and Subtract Rational Numbers
Recommended Pacing with Assessments: 11 Days

Module 4 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students added and subtracted multi-digit decimals fluently. Students used positive and negative numbers to represent quantities in real-world contexts. Students understood ordering and absolute value of rational numbers.	Students calculate the sum and difference of two integers using an algorithm. Students apply properties of operations to add and subtract rational numbers. Students solve real-world and mathematical problems involving addition and subtraction with rational numbers.	Students will solve linear equations in one variable. Students will solve pairs of linear equations.

Module 4 Vocabulary

absolute value	the distance of a number from zero on a number line; shown by ||
addend	a number added to one or more other numbers to form a sum
mixed number	a number made up of a whole number that is not zero and a fraction

Lesson 4.1 Compute Sums of Integers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Mathematical Practices and Processes

• Model with mathematics.

I Can Objective

I can identify integer addition problems. I feel confident that I can write and evaluate expressions to solve real-world integer addition problems.

Learning Objective

Calculate the sum of two integers.

Language Objective

Explain how to calculate the sum of two integers.

Vocabulary

Review: absolute value, addend

Lesson Materials

number lines

Lesson 4.2 Compute Differences of Integers

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Mathematical Practices and Processes

• Model with mathematics.

I Can Objective

I can identify integer subtraction problems. I feel confident that I can write and evaluate expressions to solve real-world integer subtraction problems.

Learning Objective

Calculate the difference of two integers without using a number line.

Language Objective

Explain how to calculate the difference of two integers.

Lesson Materials

number lines

Lesson 4.3 Compute Sums and Differences of Rational Numbers
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Attend to precision.

I Can Objective
I can identify and write expressions to represent rational number problems.

Learning Objective
Fluently add and subtract rational numbers without a number line.

Language Objective
Explain how to add and subtract rational numbers without a number line.

Vocabulary
Review: mixed number

Lesson Materials
number lines

Lesson 4.4 Apply Properties to Multi-step Addition and Subtraction Problems
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Apply properties of operations as strategies to add and subtract rational numbers.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Attend to precision.

I Can Objective
I can identify and write expressions to represent multi-step addition and subtraction problems.

Learning Objective
Use properties to solve multi-step problems involving sums and differences of positive and negative rational numbers.

Language Objective
Explain how to use properties to solve multi-step problems involving adding and subtracting rational numbers.

Unit 2: Rational Number Operations

Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 5: Multiply and Divide Rational Numbers

Recommended Pacing with Assessments: 9 Days

Module 5 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students fluently added, subtracted, multiplied, and divided multi-digit decimals using the standard algorithm for each operation.	Students develop and use the rules for multiplying and dividing signed numbers. Students solve real-world problems involving multiplication and division of integers. Students apply the rules for multiplying signed numbers to problems with three or more signed numbers. Students use properties of operations to simplify calculations.	Students will apply the properties of integer exponents to generate equivalent numerical expressions. Students will perform operations with numbers expressed in scientific notation.

Module 5 Vocabulary

Associative Property of Multiplication	the property that states that for all real numbers a, b, and c, their product is always the same regardless of their grouping
dividend	the number to be divided in a division problem
divisor	the number you are dividing by in a division problem
factor	a number you are multiplying in a multiplication problem
inverse operations	operations that undo each other: addition and subtraction, or multiplication and division
product	the result when two or more numbers are multiplied
quotient	the result when one number is divided by another
rational number	any number that can be expressed as a ratio of two integers

Lesson 5.1 Understand Multiplication and Division of Rational Number

Build Conceptual Understanding – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.

Apply properties of operations as strategies to multiply and divide rational numbers.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Model with mathematics.

I Can Objective

I can evaluate expressions and solve real-world problems involving rational number multiplication and division.

Learning Objective

Develop rules to find the product or quotient of two integers.

Language Objective

Explain how to find the product or quotient of two integers.

Vocabulary

Review: dividend, divisor, factor, inverse operations, product, quotient

Lesson Materials

two-color counters, number lines (Teacher Resource Masters)

Lesson 5.2 Multiply Rational Numbers
Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Apply properties of operations as strategies to multiply and divide rational numbers.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes
• Construct viable arguments and critique the reasoning of others.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can compute the products of signed numbers using the properties of numbers to simplify calculations.

Learning Objective
Find the product of three or more signed rational numbers.

Language Objective
Explain how to find the product of three or more signed rational numbers.

Vocabulary
Review: Associative Property of Multiplication

Lesson Materials
number lines

Lesson 5.3 Write Fractions as Decimals and Divide Integers
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\dfrac{p}{q}\right) = \dfrac{(-p)}{q} = \dfrac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.

Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Solve real-world and mathematical problems involving the four operations with rational numbers.

Mathematical Practices and Processes
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can express quotients in different forms.

Learning Objective
Express quotients in different forms.

Language Objective
Use mathematical terminology to explain how to express quotients in different forms.

Vocabulary
Review: rational number

Lesson Materials
fraction strips, Decimal Models, number lines (Teacher Resource Masters)

Lesson 5.4 Multiply and Divide Rational Numbers in Context
Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Solve real-world and mathematical problems involving the four operations
with rational numbers.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Attend to precision.

I Can Objective
I can multiply and divide rational numbers in context.

Learning Objective
Use products and quotients of rational numbers to solve problems.

Language Objective
Explain how to use products and quotients of rational numbers to solve problems.

Unit 2: Rational Number Operations
Unit 2 Project: It's Okay to Be Negative
Unit 2 Learning Mindset Focus: Strategic Help-Seeking: Identifies Need for Help

Module 6: Solve Multi-step Problems Using Rational Numbers
Recommended Pacing with Assessments and Performance Task: 9 Days

Module 6 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students performed operations with positive rational numbers. Students interpreted positive and negative numbers as opposites on a number line. Students used ordering and absolute value of rational numbers. Students added, subtracted, multiplied, and divided integers and signed rational numbers.	Students convert among forms of rational numbers as needed when solving a problem. Students solve multi-step real-world and mathematical problems involving the strategic use of operations on rational numbers. Students apply a variety of estimation strategies to check the reasonableness of answers to real-world problems.	Students will apply the properties of integer exponents to generate equivalent numerical expressions. Students will perform operations with numbers expressed in scientific notation. Students will establish connections among proportional relationships, lines, and linear equations.

Module 6 Vocabulary

compatible numbers	numbers that are close to the given numbers that make estimation or mental calculation easier
Distributive Property	the property that states that for all real numbers a, b, and c, $a(b + c) = ab + ac$, and $a(b - c) = ab - ac$
Identity Property of Multiplication	the property that states that the product of 1 and any number is that number
Inverse Property of Multiplication	the property that states that the product of any number and its reciprocal, or multiplicative inverse, is 1
order of operations	a rule for evaluating expressions: first perform the operations in parentheses, then compute powers or roots, then perform all multiplication and division from left to right, and then perform all addition and subtraction from left to right

Lesson 6.1 Apply Properties and Strategies to Operate with Rational Numbers

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving the four operations with rational numbers.

Apply properties of operations as strategies to multiply and divide rational numbers.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can write and evaluate expressions involving rational numbers and name the properties used.

Learning Objective

Apply properties and strategies to operate with rational numbers.

Language Objective

Explain how to apply properties and strategies to operate with rational numbers.

Vocabulary

Review: Identity Property of Multiplication, Inverse Property of Multiplication, order of operations

Lesson 6.2 Estimate to Check Reasonableness

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can underestimate and overestimate to check the reasonableness of answers.

Learning Objective

Use estimation to check the reasonableness of answers when solving multi-step real-world problems.

Language Objective

Explain strategies of estimation and say whether an estimate is likely to be an underestimate or an overestimate.

Vocabulary

Review: compatible numbers, Distributive Property

Lesson 6.3 Solve Multi-step Problems with Rational Numbers in Context

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Use proportional relationships to solve multistep ratio and percent problems.

Mathematical Practices and Processes

• Attend to precision.

I Can Objective

I can solve multi-step problems that involve rational numbers in different forms and multiple operations.

Learning Objective

Solve multi-step problems involving a combination of rational number operations.

Language Objective

Use terminology related to rational numbers and operations to solve multi-step problems and justify the reasonableness of answers.

Unit 3: Model with Expressions, Equations, and Inequalities

Unit 3 Project: The Rhind Papyrus
Unit 3 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 7: Solve Problems Using Expressions and Equations

Recommended Pacing with Assessments: 11 Days

Module 7 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students solved real-world problems by writing and solving equations of the form $x + p = q$ and $px = q$, where p, q and x are all nonnegative rational numbers.	Students add, subtract, factor, and expand linear expressions with rational coefficients. Students rewrite expressions to shed light on a problem and how the quantities in it are related. Students construct simple equations and inequalities to solve problems by reasoning about the quantities. Students write and solve equations for an unknown angle in a figure.	Students will know and apply the properties of integer exponents to generate equivalent numerical expressions. Students will use square-root and cube-root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Students will solve linear equations in one variable. Students will analyze two-variable linear equations and pairs of simultaneous equations.

Module 7 Vocabulary

Associative Property of Addition	the property that states that for all real numbers a, b, and c, the sum is always the same, regardless of their grouping
Commutative Property of Addition	the property that states that two or more numbers can be added in any order without changing the sum
Division Property of Equality	the property that states that if you divide both sides of an equation by the same nonzero number, the new equation will have the same solution
equilateral triangle	a triangle with three congruent sides
greatest common factor	the largest common factor of two or more numbers
isosceles triangle	a triangle with at least two congruent sides
like terms	terms with the same variables raised to the same exponents

perimeter	the distance around a polygon
right angle	an angle that measures 90°
solution of an equation	a value or values that make an equation true
terms (of an expression)	the parts of an expression that are added or subtracted
adjacent angle	angles in the same plane that have a common vertex and a common side
complementary angles	angles whose measures have a sum of 90°
supplementary angles	two angles whose measures have a sum of 180°
vertical angles	a pair of opposite, congruent angles formed by intersecting lines

Lesson 7.1 Write Linear Expressions in Different Forms for Situations

Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Model with mathematics.

I Can Objective
I can write different forms of linear expressions to represent the same real-world situation.

Learning Objective
Use linear expressions to represent a quantity in different ways.

Language Objective
Write equivalent expressions to represent a real-world situation and use the properties of operations for real numbers to transform expressions.

Lesson Materials
counters

Lesson 7.2 Add, Subtract, and Factor Linear Expressions with Rational Coefficients

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities are related.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Attend to precision.

I Can Objective
I can identify problems that require me to add, subtract, factor, and expand linear expressions with rational coefficients.

Learning Objective
Add, subtract, factor and expand linear expressions with rational coefficients.

Language Objective
Write expressions to model real-world situations and then simplify those expressions by using the commutative property of addition and the distributive property. Rewrite simplified expressions into factored form using the distributive property.

Vocabulary
Review: Associative Property of Addition, Commutative Property of Addition, equilateral triangle, greatest common factor, like terms, term

Lesson 7.3 Write Two-Step Equations for Situations

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices and Processes
• Model with mathematics.
• Look for and make use of structure.

I Can Objective
I can write two-step equations for various situations.

Learning Objective
Represent a real-world situation with an equation.

Language Objective
Write equations of the form $px + q = r$ and $p(x + q) = r$ to represent real-world situations.

Vocabulary
Review: isosceles triangle, perimeter

Lesson 7.4 Apply Two-Step Equations to Solve Real-World Problems

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.

I Can Objective

I can use two-step equations to solve a variety of problems.

Learning Objective

Solve real-world situations using an equation.

Language Objective

Solve real-world situations that can be modeled with equations of the form $px + q = r$ and $p(x + q) = r$.

Vocabulary

Review: Division Property of Equality, solution of an equation

Lesson Materials

algebra tiles

Lesson 7.5 Apply Two-Step Equations to Find Angle Measures
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Mathematical Practices and Processes
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can identify angle relationships, and use them to write and solve equations.

Learning Objective
Write and solve two-step equations involving unknown angle measurements.

Language Objective
Identify and describe pairs of angles that are complementary, supplementary, vertical, and adjacent.

Vocabulary
Review: right angle
New: adjacent angles, complementary angles, supplementary angles, vertical angles

Unit 3: Model with Expressions, Equations, and Inequalities

Unit 3 Project: The Rhind Papyrus
Unit 3 Learning Mindset Focus: Resilience: Identifies Obstacles

Module 8: Solve Problems Using Inequalities

Recommended Pacing with Assessments and Performance Task: 11 Days

Module 8 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students wrote inequalities of the form $x > c$ or $x < c$ to represent constraints or conditions in a real-world or mathematical problem. Students solved equations as a process of answering a question. Students used variables to represent quantities in a real-world or mathematical problem, and constructed simple equations to solve problems by reasoning about the quantities.	Students solve one- and two-step inequalities for real-world problems. Students write and solve inequalities in the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Students graph the solution sets of inequalities and interpret them in the context of the problem.	Students will solve compound inequalities. Students will graph and solve inequalities in two variables.

Module 8 Vocabulary

equivalent	having the same value
inequality	a mathematical sentence that shows the relationship between quantities that are not equivalent
solution of an inequality	a value or values that make an inequality true

Lesson 8.1 Understand and Apply Properties to Solve One-Step Inequalities

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical Practices and Processes
• Look for and make use of structure.
• Reason abstractly and quantitatively.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can solve one-step inequalities by applying properties and graph the solution on a number line.

Learning Objective
Apply properties to solve one-step inequalities.

Language Objective
Explain how to set up and solve one-step inequalities.

Vocabulary
Review: equivalent, inequality, solution of an inequality

Lesson Materials
number lines (Teacher Resource Masters)

Lesson 8.2 Write Two-Step Inequalities for Situations

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices and Processes
• Model with mathematics.
• Look for and make use of structure.

I Can Objective
I can write two-step inequalities for real-world scenarios and determine whether given values are solutions.

Learning Objective
Write two-step inequalities to represent situations.

Language Objective
Use the terms *greater than*, *less than*, *greater than or equal to*, and *less than or equal to* in the context of inequalities to represent situations.

Lesson Materials
number lines (Teacher Resource Masters)

Lesson 8.3 Apply Two-Step Inequalities to Solve Problems

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective

I can write and solve two-step inequalities for real-world scenarios.

Learning Objective

Write and solve two-step inequalities to solve problems.

Language Objective

Write inequalities to represent situations and interpret the solutions in context.

Unit 4: Geometry
Unit 4 Project: Buffon's Needle
Unit 4 Learning Mindset Focus: Perseverance: Learns Effectively

Module 9: Draw and Analyze Two-Dimensional Figures
Recommended Pacing with Assessments: 10 Days

Module 9 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students identified and drew lines of symmetry. Students found areas of figures by decomposing those figures into triangles. Students found volumes of right rectangular prisms. Students found surface areas of figures using nets. Students drew polygons in the coordinate plane. Students used angle relationships to solve problems.	Students draw and construct circles and other figures with given conditions using technology and freehand. Students analyze how many, if any, triangles can be made from given side lengths or angle measures.	Students will verify the properties of rotations, reflections, and translations. Students will understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Students will use informal arguments to establish facts about the angle sum and exterior angle of triangles theorems, about the angles constructed when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Module 9 Vocabulary

parallel	word used to describe lines that never intersect or segments that are parts of parallel lines
perpendicular	intersecting to form a right angle
diameter	a line segment that passes through the center of a circle and has endpoints on the circle, or the length of that segment
radius	a line segment with one endpoint at the center of a circle and the other endpoint on the circle, or the length of that segment

Lesson 9.1 Draw Circles and Other Figures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

• Use appropriate tools strategically.
• Attend to precision.

I Can Objective

I can inscribe triangles in circles and draw geometric figures meeting given conditions.

Learning Objective

Draw and construct circles and other figures using technology and freehand with given conditions.

Language Objective

Describe circles and other figures using the words *circumference*, *radius*, *diameter*, *symmetry*, *parallel*, and *perpendicular*.

Vocabulary

Review: parallel, perpendicular
New: diameter, radius

Lesson Materials

ruler, compass, protractor

Lesson 9.2 Draw and Construct Triangles Given Side Lengths

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes

• Construct viable arguments and critique the reasoning of others.
• Use appropriate tools strategically.

I Can Objective

I can determine whether three lengths could be side lengths of a triangle, and, given two side lengths, I can find the range of possible lengths for the third side.

Learning Objective

Determine how many triangles or quadrilaterals can be made given the side lengths: none, one, or many.

Language Objective

Understand and be able to explain whether you can form no triangle, one triangle, or many triangles from three given side lengths. Understand and be able to explain whether you can form no quadrilateral, one quadrilateral, or many quadrilaterals from four given side lengths.

Lesson Materials

ruler, compass

Lesson 9.3 Draw and Construct Triangles Given Angle Measures

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes
• Use appropriate tools strategically.

I Can Objective
I can determine whether it is possible to draw a triangle with three given angle measures. If it is, I can construct such a triangle.

Learning Objective
Determine how many triangles can be made given the angle measures: none, one, or many.

Language Objective
Describe angle measures and the relationship they form in a triangle.

Lesson Materials
ruler, protractor, compass, grid paper, geometry software

Lesson 9.4 Draw and Analyze Shapes to Solve Problems

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Use appropriate tools strategically.

I Can Objective
I can use tools to draw or construct figures that meet given criteria, and I can analyze the resulting figures.

Learning Objective
Draw, construct, and analyze two-dimensional figures to solve real-world problems.

Language Objective
Use the terms *center* and *radius* to explain the steps in constructing a side with a given length.

Lesson Materials
grid paper, ruler, protractor, compass

Unit 4: Geometry
Unit 4 Project: Buffon's Needle
Unit 4 Learning Mindset Focus: Perseverance: Learns Effectively

Module 10: Analyze Figures to Find Circumference and Area
Recommended Pacing with Assessments: 9 Days

Module 10 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students calculated circumference and area of circles.	Students use the formulas for circumference to solve problems.	Students will derive the equation of a circle using the Pythagorean Theorem.
Students found surface area and volume of solids.	Students describe cross sections of circular solids that result in circles, rectangles, and triangles.	Students will solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Students found areas of triangles and special quadrilaterals.	Students calculate the areas and circumferences of cross sections.	
Students found the diameter and radius of circles.	Students calculate the area of a composite figure.	Students will calculate arc length, sector area, and analyze inscribed & circumscribed circles.
	Students solve multi-step real-life and mathematical problems.	

Module 10 Vocabulary

base	a side of a polygon
height	in a quadrilateral, the perpendicular distance from the base to the opposite vertex or side
circumference	the distance around a circle for one rotation
cross section	a two-dimensional figure formed by the intersection of a plane and a solid figure
pi (π)	the ratio of the circumference of a circle to the length of its diameter; $\pi \approx 3.14$ or $\frac{22}{7}$
plane	a flat surface that has no thickness and extends forever

Lesson 10.1 Derive and Apply Formulas for Circumference

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Mathematical Practices and Processes

• Model with mathematics.
• Look for and make use of structure.

I Can Objective

I can use the circumference formulas $C = \pi d$ and $C = 2\pi r$ to solve for C, r, or d when the value of the other variable is given.

Learning Objective

Derive and apply formulas for circumference.

Language Objective

Explain how to derive and apply formulas for circumference.

Vocabulary

New: circumference, pi (π)

Lesson Materials

compass, ruler, string, circular objects

Lesson 10.2 Derive and Apply a Formula for the Area of a Circle

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.

I Can Objective

I can use the area formulas for a circle to find the area of a circle if I know its radius or circumference or to find the radius or circumference if I know the area.

Learning Objective

Derive and apply formulas for the area of a circle.

Language Objective

Explain the meaning of the terms *radius*, *diameter*, *circumference*, and *area* of circles.

Vocabulary

Review: base, height

Lesson Materials

compass, grid paper

Lesson 10.3 Describe and Analyze Cross Sections of Circular Solids
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes
• Look for and make use of structure.

I Can Objective
I can identify the shapes of cross sections of circular solids and solve problems involving the areas of cross sections.

Learning Objective
Describe and analyze cross sections of circular solids that result in circles, rectangles, and triangles.

Language Objective
Describe and analyze cross sections of circular solids comparing sides of cross sections to the radius, diameter, and height of solids.

Vocabulary
New: cross section, plane

Lesson Materials
ruler

Lesson 10.4 Areas of Composite Figures
Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes
• Look for and make use of structure.

I Can Objective
I can break a composite figure into simple shapes and use area formulas to find its area.

Learning Objective
Use known formulas to calculate the areas of composite figures.

Language Objective
Describe the area of a composite figure as an assemblage of its component features.

Unit 4: Geometry
Unit 4 Project: Buffon's Needle
Unit 4 Learning Mindset Focus: Perseverance: Learns Effectively

Module 11: Analyze Surface Area and Volume
Recommended Pacing with Assessments and Performance Task: 10 Days

Module 11 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students described two-dimensional figures (circles, triangles, and rectangles) created by slicing a three-dimensional circular solid. Students used nets to find the surface area of three-dimensional figures. Students found the volume of prisms. Students solved real-world and mathematical problems involving area of two-dimensional composite figures.	Students describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and pyramids. Students solve real-world and mathematical problems involving volume and surface area of three-dimensional objects.	Students will identify the shapes of two-dimensional cross sections of three-dimensional objects and three-dimensional objects generated by rotations of two-dimensional objects. Students will find surface area of cylinders, pyramids, cones, and spheres. Students will find volumes of spheres, cones, and cylinders and use them to solve problems.

Module 11 Vocabulary

cross section	a two-dimensional figure formed by the intersection of a plane and a solid
net	an arrangement of two-dimensional figures that can be folded to form a solid figure
pyramid	a polyhedron with a polygon base and triangular sides that all meet at a common vertex
rectangular prism	a polyhedron whose bases are rectangles and whose other faces are parallelograms
surface area	the sum of the areas of the faces, or surfaces, of a three-dimensional figure
vertex	on an angle or polygon, the point where two sides intersect
volume	the number of cubic units needed to fill a given space

Lesson 11.1 Describe and Analyze Cross Sections of Prisms and Pyramids

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Mathematical Practices and Processes
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can describe and analyze the cross sections of pyramids and prisms of all types, with or without a diagram.

Learning Objective
Identify and describe the two-dimensional figures resulting from horizontal and vertical cross sections of pyramids and prisms.

Language Objective
Use the properties of two-dimensional figures to correctly identify and name cross sections.

Vocabulary
Review: cross section, pyramid, rectangular prism, vertex

Lesson Materials
ruler; Square Pyramid Net, grid paper (Teacher Resource Masters)

Lesson 11.2 Derive and Apply Formulas for Surface Areas of Cubes and Right Prisms

Connect Concepts and Skills – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes
• Model with mathematics.

I Can Objective
I can derive and apply the formulas for surface area of any right prism.

Learning Objective
Learn to calculate the surface area of a right prism using the surface area formula.

Language Objective
Derive and explain how to use the surface area formula.

Vocabulary
Review: net, surface area

Lesson Materials
ruler; Rectangular Prism Net, grid paper (Teacher Resource Masters)

Lesson 11.3 Derive and Apply a Formula for the Volume of a Right Prism

Connect Concepts and Skills –1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Model with mathematics.
• Use appropriate tools strategically.

I Can Objective

I can accurately apply the formula to find the volume of right prisms.

Learning Objective

Calculate the volume of a right prism using the volume formula.

Language Objective

Derive and explain how to use the volume formula.

Vocabulary

Review: volume

Lesson Materials

centimeter cubes, base-ten blocks;
grid paper (Teacher Resource Masters)

Lesson 11.4 Solve Multi-step Problems with Surface Area and Volume

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Model with mathematics.
• Attend to precision.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can solve multi-step surface area and volume problems.

Learning Objective

Solve multi-step problems involving three-dimensional figures using formulas for surface area and volume.

Language Objective

Explain how to use the surface area formula to find volume and the volume formula to find surface area.

Unit 5: Sampling and Data Analysis
Unit 5 Project: A Birthday Puzzle
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 12: Proportional Reasoning with Samples
Recommended Pacing with Assessments: 7 Days

Module 12 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students explored statistical data collection. Students learned how to choose a representative sample. Students analyzed and graphed statistical data.	Students understand how to select a representative sample. Students understand the ways that a sample could be biased. Students understand that proportional reasoning can be used to make inferences about a population from a random sample. Students use proportional reasoning to make inferences about a population. Students use data from a random sample to draw inferences about a population.	Students will construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Students will describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Module 12 Vocabulary

bias	when a sample does not accurately represent the population
population	the entire group of objects or individuals considered for a survey
random sample	a sample in which each individual or object in the entire population has an equal chance of being selected
representative sample	a sample that has the same characteristics of the population
sample	a part of the population

Lesson 12.1 Understand Representative Samples
Build Conceptual Understanding – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.

I Can Objective
I can identify the population and sample of given survey scenarios, and determine whether the sample is random and/or representative of the population.

Learning Objective
Understand populations, random samples, and how to select a representative sample.

Language Objective
Describe samples as a subset of a population, and analyze samples to determine whether they are representative of a population.

Vocabulary
New: bias, population, random sample, representative sample, sample

Lesson 12.2 Make Inferences from a Random Sample
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Attend to precision.
• Look for and express regularity in repeated reasoning.

I Can Objective
I can use proportional reasoning to make inferences about populations based on the results of a random sample.

Learning Objective
Use a random sample to make inferences about a population.

Language Objective
Use a random sample to describe an inference or prediction about a population after setting up and solving a proportion based on the sample.

Lesson 12.3 Make Inferences from Repeated Random Samples
Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Mathematical Practices and Processes
• Attend to precision.
• Construct viable arguments and critique the reasoning of others.

I Can Objective
I can calculate sample ratios, and I can make inferences about the populations from the samples.

Learning Objective
Understand that repeatedly sampling a population with the same size random sample will cause the data to vary.

Language Objective
Analyze repeated samples taken from a population with a known population proportion. Find sample proportions and verbally quantify the variation between the population and the samples.

Lesson Materials
pieces of red, blue, and green paper

Unit 5: Sampling and Data Analysis

Unit 5 Project: A Birthday Puzzle
Unit 5 Learning Mindset Focus: Resilience: Manages the Learning Process

Module 13: Use Statistics and Graphs to Compare Data

Recommended Pacing with Assessments and Performance Task: 9 Days

Module 13 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students displayed data in dot plots and box plots. Students calculated five key values: minimum, lower quartile, median, upper quartile, and maximum. Students learned about random samples, measures of center, and measures of variability.	Students compare the shapes, centers, and spreads of data displayed in dot plots and box plots. Students draw conclusions about populations based on sample data displayed in dot plots and box plots. Students use measures of center and variability, including mean absolute deviation for data from random samples to draw informal comparisons about the populations.	Students will investigate bivariate data using scatter plots and looking for patterns of association. Students will informally fit a straight line to a scatter plot and informally assess the model fit. Students will use the equation of a linear model to solve problems in the context of bivariate measurement data. Students will understand that patterns of association can be seen in bivariate categorical data.

Module 13 Vocabulary

interquartile range	the difference between the upper and lower quartiles in a box plot
lower quartile	the median of the lower half of a set of data
mean	the sum of the items in a set of data divided by the number of items in the set; also called *average*
mean absolute deviation	the mean distance between each data value and the mean of the data set
median	the middle number, or the mean (average) of the two middle numbers, in an ordered set of data
range	the difference between the greatest and least values in a data set
upper quartile	the median of the upper half of a set of data

Lesson 13.1 Compare Center and Spread of Data Displayed in Dot Plots

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.

I Can Objective
I can compare important characteristics of data sets displayed in dot plots to draw conclusions based on data.

Learning Objective
Compare the center and spread of data displayed in dot plots.

Language Objective
Use the terms *center* and *spread* to describe key features of data sets displayed in dot plots.

Vocabulary
Review: mean, median, range

Lesson 13.2 Compare Center and Spread of Data Displayed in Box Plots

Apply and Practice – 1 Day

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective
I can make box plots and draw inferences about populations based on displayed data.

Learning Objective
Compare data displayed in box plots, and use these comparisons to draw inferences about two populations.

Language Objective
Use the terms *box plot*, *range*, and *interquartile range* when comparing key features of data sets.

Vocabulary
Review: interquartile range, lower quartile, upper quartile

Lesson 13.3 Compare Means Using Mean Absolute Deviation and Repeated Sampling
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective
I can use the means and MADS to assess the amount of visual overlap of two numerical data distributions.

Learning Objective
Use means and MADs to compare two populations.

Language Objective
Use the terms *mean absolute deviation* and *mean* to compare differences in center and spread of two populations.

Vocabulary
Review: mean absolute deviation

Unit 6: Probability
Unit 6 Project: Class Arcade
Unit 6 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 14: Understand and Apply Experimental Probability
Recommended Pacing with Assessments: 10 Days

Module 14 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students summarized numerical data sets in relation to their context. Students identified and represented proportional relationships.	Students conduct experiments to infer probabilities and rank outcomes by likelihood. Students represent sample spaces for compound events. Students design and use simulations to generate frequencies for simple and compound events. Students approximate the probability of a chance event by collecting data and predict the approximate relative frequency given the probability.	Students will understand and apply theoretical probability. Students will calculate theoretical probabilities of compound events and understand conditional probability. Students will construct and interpret two-way frequency tables and two-way relative frequency tables.

Module 14 Vocabulary

complement	the set of all outcomes in the sample space that are not included in the event
compound event	an event made up of two or more simple events
event	an outcome or set of outcomes of an experiment or situation
experiment	in probability, any activity based on chance in which results are observed
experimental probability	the ratio of the number of times an event occurs to the total number of trials, or times that the activity is performed
outcome	a possible result of a probability experiment
probability	a number from 0 to 1 (or 0% to 100%) that describes how likely an event is to occur
probability of an event	the probability of an event is the ratio of the number of outcomes in the event to the total number of outcomes in the sample space
proportion	an equation that states that two ratios are equivalent
sample space	all possible outcomes of an experiment
simulation	a model of an experiment, often one that would be too difficult or too time-consuming to actually perform
trial	each repetition or observation of an experiment

Lesson 14.1 Understand Probability of an Event
Build Conceptual Understanding – 1 Day

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective
I can describe the likelihood of an event.

Learning Objective
Describe the likelihood of an event in terms of a probability between 0 and 1.

Language Objective
Use probability, a number between 0 and 1 (or 0% and 100%), to describe the likelihood of an event, with 0 representing an impossible event, and 1 representing a certain event.

Vocabulary
New: event, experiment, outcome, probability, probability of an event, sample space, trial

Lesson Materials
number cubes, coins

Lesson 14.2 Find Experimental Probability of Simple Events
Connect Concepts and Skills – 2 Days

Conceptual	**Conceptual and Procedural**	**Procedural**
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Use appropriate tools strategically.

I Can Objective
I can find an experimental probability and its complement.

Learning Objective
Find the experimental probability of an event.

Language Objective
Describe how to find the experimental probability of an event. Explain how to find the complement of the experimental probability of an event.

Vocabulary
New: complement, experimental probability, simulation

Lesson Materials
paper cups, coins, number cards 0-9

Lesson 14.3 Find Experimental Probability of Compound Events
Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Design and use a simulation to generate frequencies for compound events.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Reason abstractly and quantitatively.
• Use appropriate tools strategically.
• Look for and make use of structure.

I Can Objective
I can find the experimental probability of a compound event.

Learning Objective
Determine the probability of compound events.

Language Objective
Use the terms *compound event* and *simulation* to explain the solutions to real-world probability problems.

Vocabulary
New: compound event

Lesson 14.4 Use Experimental Probability and Proportional Reasoning to Make Predictions

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Use proportional relationships to solve multi-step ratio and percent problems.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Model with mathematics.
• Attend to precision.

I Can Objective

I can use proportional reasoning or percent expressions to make a prediction based on an experimental probability.

Learning Objective

Use experimental probability and proportional reasoning to make predictions about real-world scenarios.

Language Objective

Explain the meaning of proportions and percent equations and how to use them to predict the frequency of long-term events.

Vocabulary

New: proportion

Unit 6: Probability
Unit 6 Project: Class Arcade
Unit 6 Learning Mindset Focus: Challenge-Seeking: Defines Own Challenges

Module 15: Understand and Apply Theoretical Probability
Recommended Pacing with Assessments and Performance Task: 13 Days

Module 15 Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students approximated the probability of a chance event by collecting data and predicted the approximate relative frequency given the probability. Students developed a probability model by observing frequencies in chance process data. Students found the experimental probability of a compound event. Students developed an understanding of statistical variability.	Students compare probabilities from a model to observed frequencies. Students develop a uniform probability model by assigning equal probability to all outcomes and use it to determine probabilities of events. Students calculate theoretical probabilities of compound events. Students design and use a simulation to generate frequencies for compound events.	Students will investigate patterns of association in bivariate data. Students will construct and interpret two-way frequency tables and two-way relative frequency tables.

Module 15 Vocabulary

tree diagram	a branching diagram that shows all possible combinations or outcomes of an event
theoretical probability	the ratio of the number of possible outcomes in the event to the total number of possible outcomes in the sample space

Lesson 15.1 Find Theoretical Probability of Simple Events

Connect Concepts and Skills – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes

• Reason abstractly and quantitatively.
• Look for and make use of structure.

I Can Objective

I can find the theoretical probability of a simple event.

Learning Objective

Find the theoretical probability of simple events and compare theoretical probability to experimental probability.

Language Objective

Interpret the context of simple events to find the theoretical probability.

Vocabulary

New: theoretical probability

Lesson Materials

coins

Lesson 15.2 Find Theoretical Probability of Compound Events
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Look for and make use of structure.

I Can Objective
I can find the theoretical probability of a compound event.

Learning Objective
Find and compare theoretical probabilities of compound events using a table, a tree diagram, and an organized list.

Language Objective
Interpret the context of compound events to find the theoretical probability.

Vocabulary
Review: tree diagram

Lesson 15.3 Use Theoretical Probability and Proportional Reasoning to Make Predictions

Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probability of events.

Use proportional relationships to solve multi-step ratio and percent problems.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematical Practices and Processes

• Model with mathematics.
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.

I Can Objective

I can use theoretical probability to make predictions about real-world situations.

Learning Objective

Use theoretical probability and proportional reasoning to make a prediction about a simple or compound event, and make a qualitative prediction.

Language Objective

Make qualitative predictions about an event, and describe reasons for any discrepancies.

Lesson 15.4 Conduct Simulations
Apply and Practice – 2 Days

Conceptual	Conceptual and Procedural	Procedural
Build Conceptual Understanding	Connect Concepts and Skills	Apply and Practice

Mathematics Standards
Design and use a simulation to generate frequencies for compound events.

Use proportional relationships to solve multi-step ratio and percent problems.

Mathematical Practices and Processes
• Use appropriate tools strategically.
• Attend to precision.

I Can Objective
I can use a simulation to test the probability of simple and compound events.

Learning Objective
Design and perform a simulation to test the probability of a simple event or a compound event.

Language Objective
Interpret the context of a real-world scenario to determine the best simulation model to test the probability of a simple or compound event.

Lesson Materials
slips of paper